Challenges for numerical relativity and gravitational-wave source modeling

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JOHNS HOPKINS UNIVERSITY

Focus of this talk: what improvements in numerical relativity do we need to test GR with binary black hole mergers?

"The binary black hole problem has been **solved** 15 years ago"

In general relativity, for
comparable-mass
nonspinning
nonprecessing
noneccentric
binaries

(and I won't talk about neutron stars…)

What keeps me up at night:

Systematic errors in GR: not good enough for LISA/CE/ET

- spectroscopy tests
- sky localization

Identifying “best” beyond-GR theories:

- specific theories vs. parametrization

Punchline: NR may guide theoretical work

Group







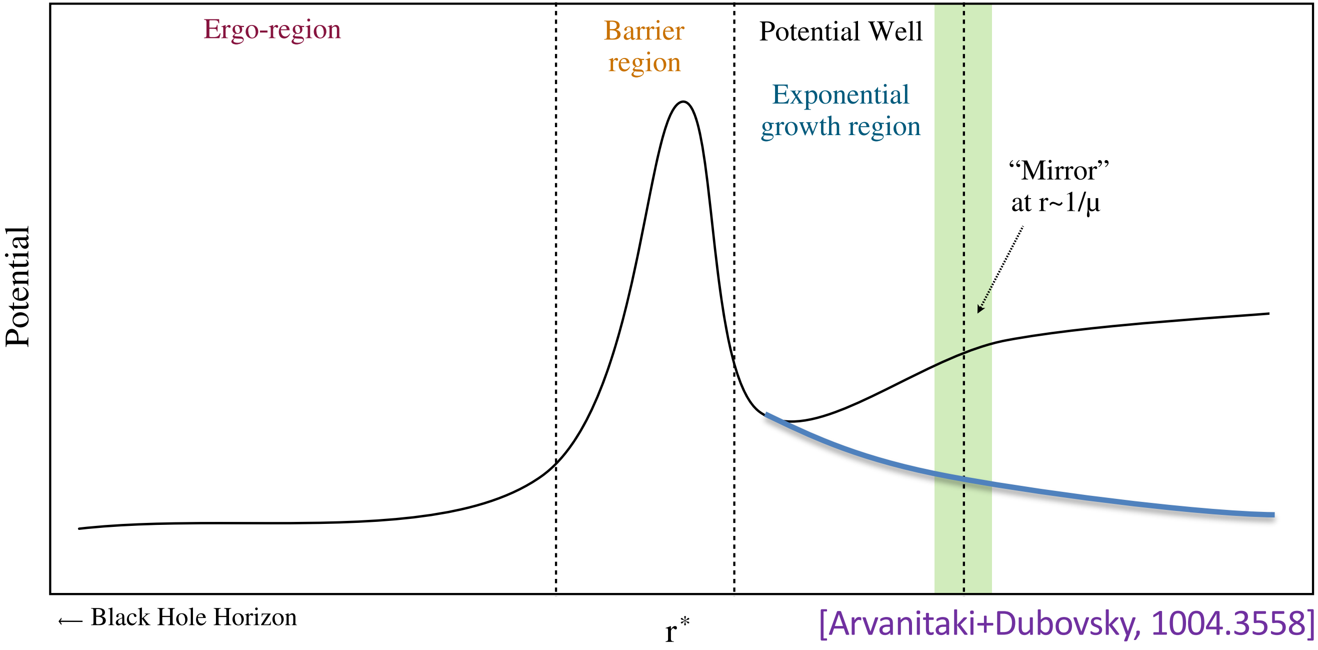
GWIC prize 2016

GWIC prize 2017

Black hole spectroscopy: a null test

Quasinormal (and superradiant) modes

$$\left(\frac{d^2}{dr_*^2}+\omega^2\right)\Phi = V\Phi$$



[Arvanitaki+Dubovsky, 1004.3558]

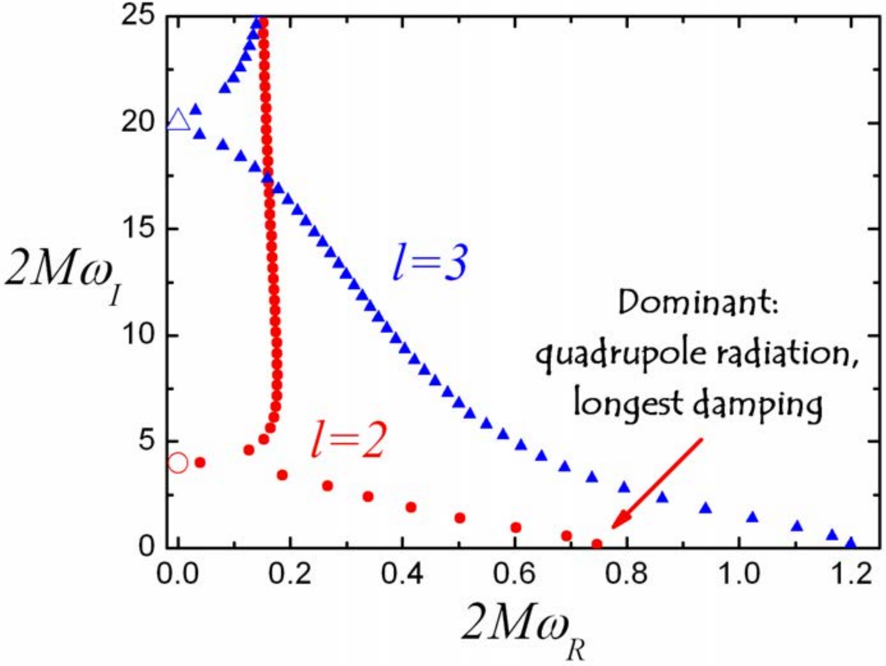
Quasinormal modes:

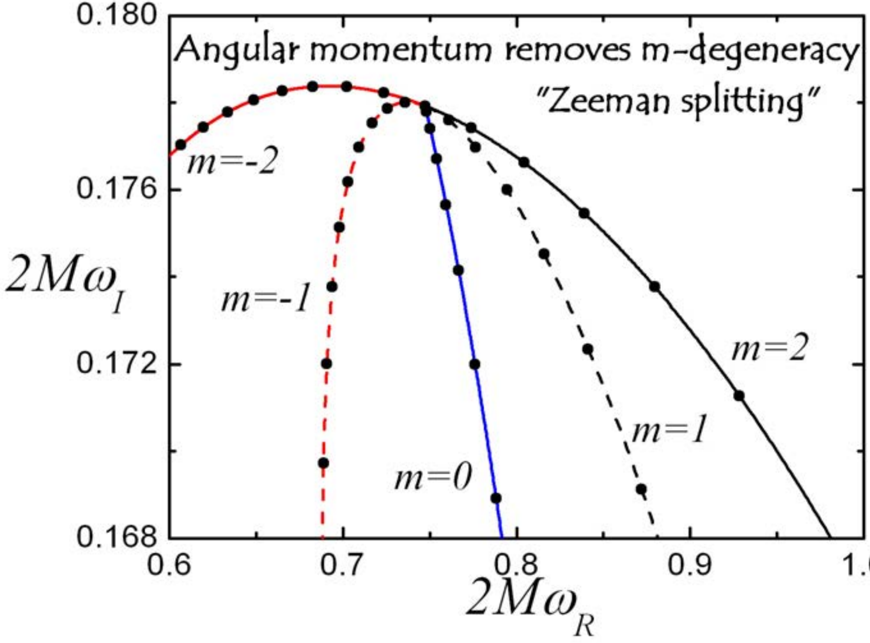
- Ingoing waves at the horizon, outgoing waves at infinity
- Spectrum of damped modes ("ringdown") [EB+, 0905.2975]

Massive scalar field:

- Superradiance: black hole bomb when $0 < \omega < m\Omega_H$ [Press-Teukolsky 1972]
- Hydrogen-like, unstable bound states [Detweiler 1980, Zouros+Eardley, Dolan…]

Schwarzschild and Kerr quasinormal mode spectrum

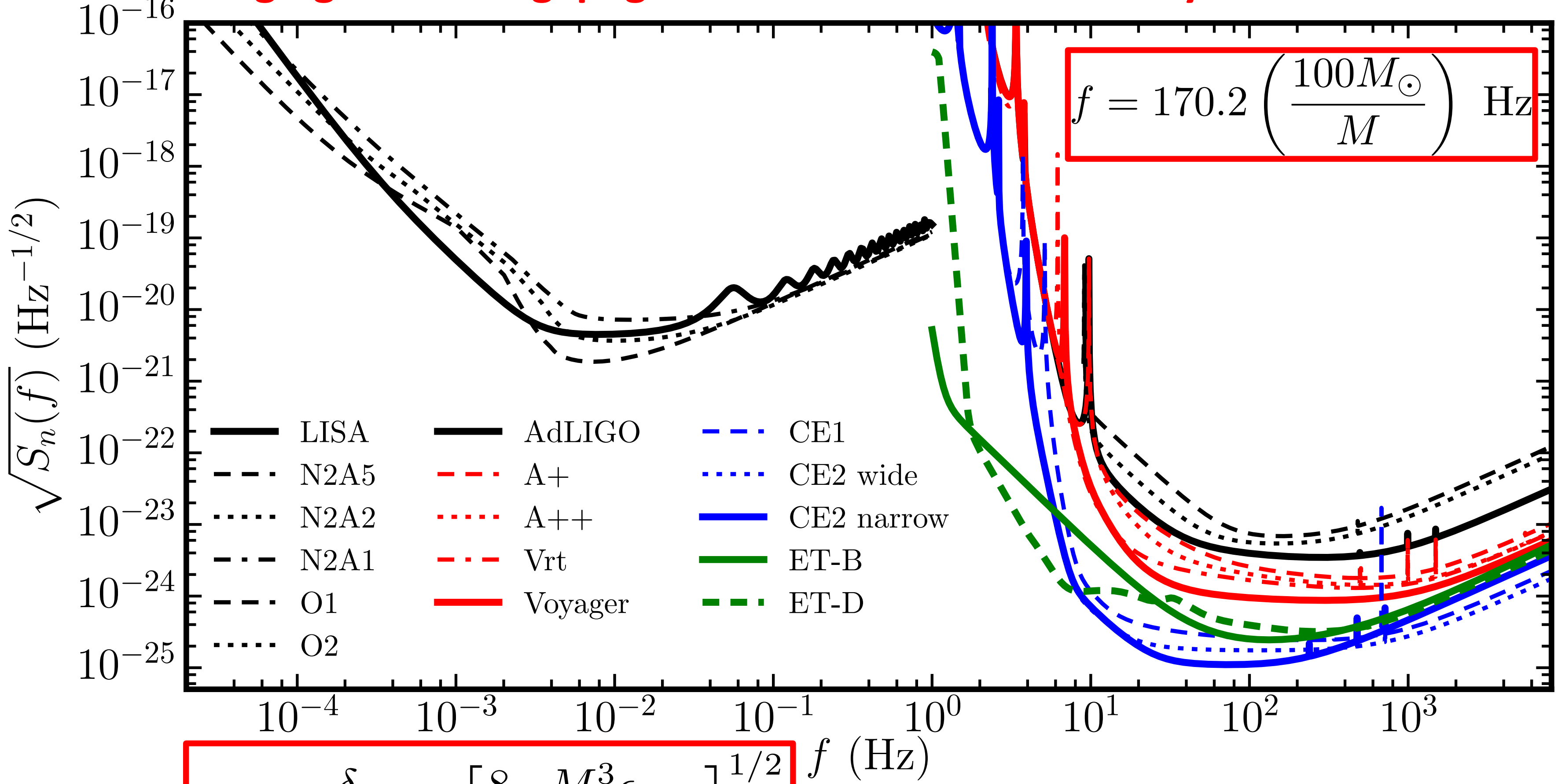




- **One mode** fixes mass and spin – and the whole spectrum!
- **N modes:** N tests of GR dynamics…**if** they can be measured
- Needs SNR>50 or so for a comparable mass, nonspinning binary merger

[Berti-Cardoso-Will, gr-qc/0512160; EB+, gr-qc/0707.1202]

Bridging the mass gap: gravitational wave astronomy in the 2030s

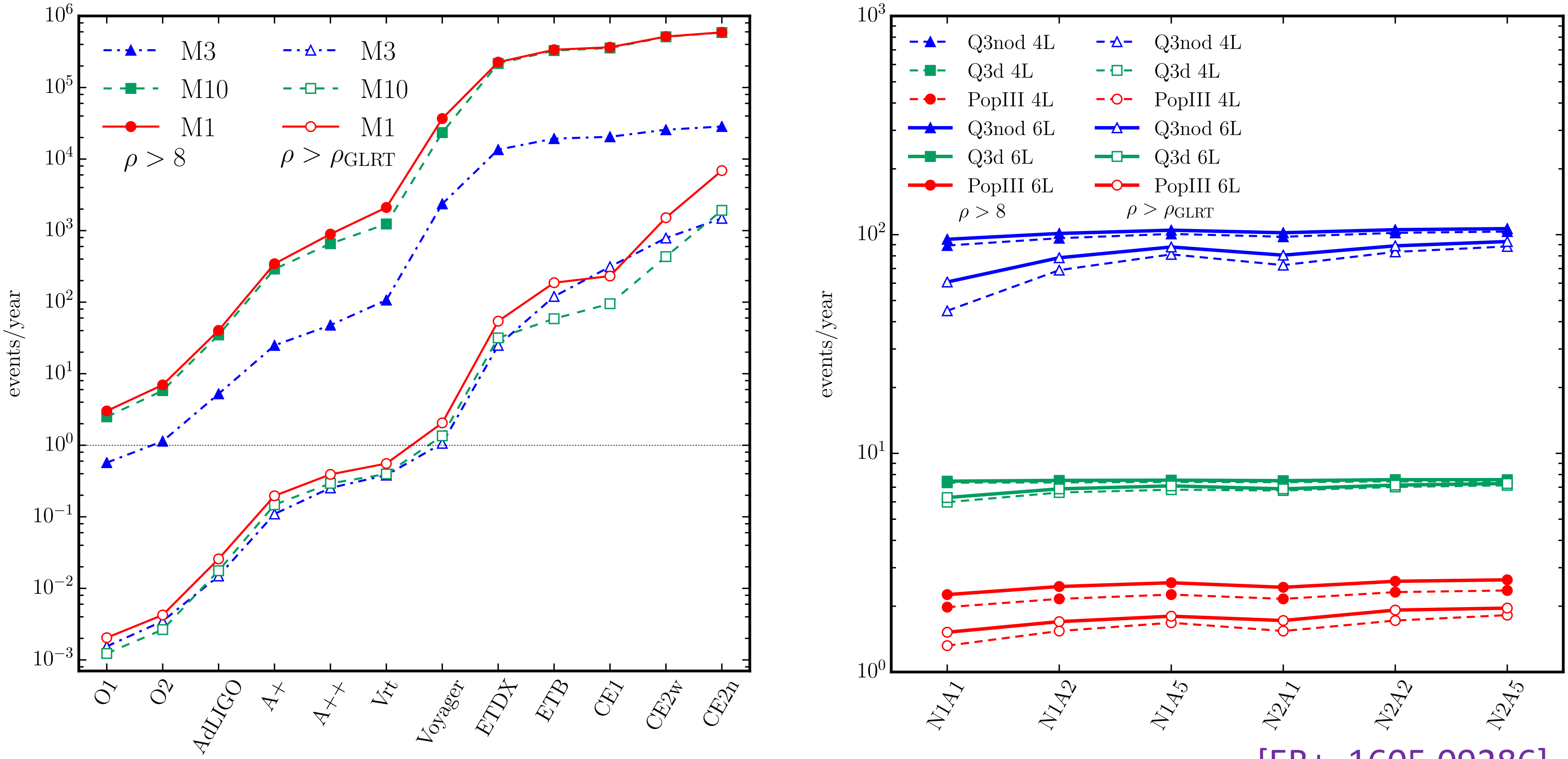


$$f = 170.2\left(\frac{100M_\odot}{M}\right)\ \mathrm{Hz}$$

$$\rho = \frac{\delta_{\rm eq}}{D_{\rm L}\mathcal{F}_{lmn}}\left[\frac{8}{5}\frac{M_z^3\epsilon_{\rm rd}}{S_n(f_{lmn})}\right]^{1/2}$$

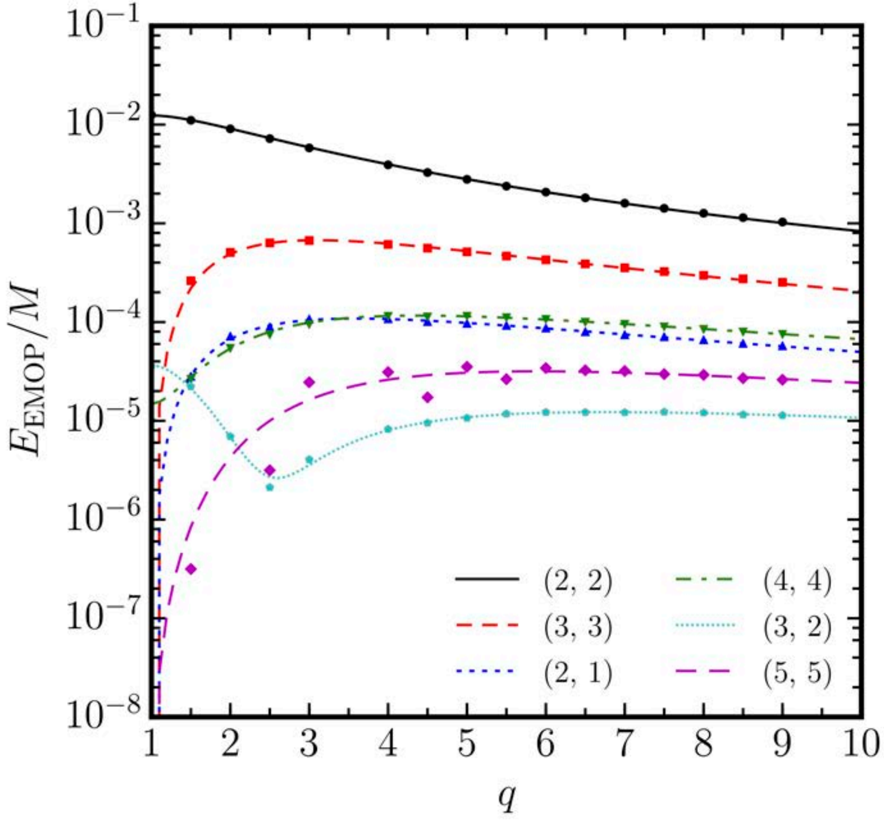
[EB+, 1605.09286]

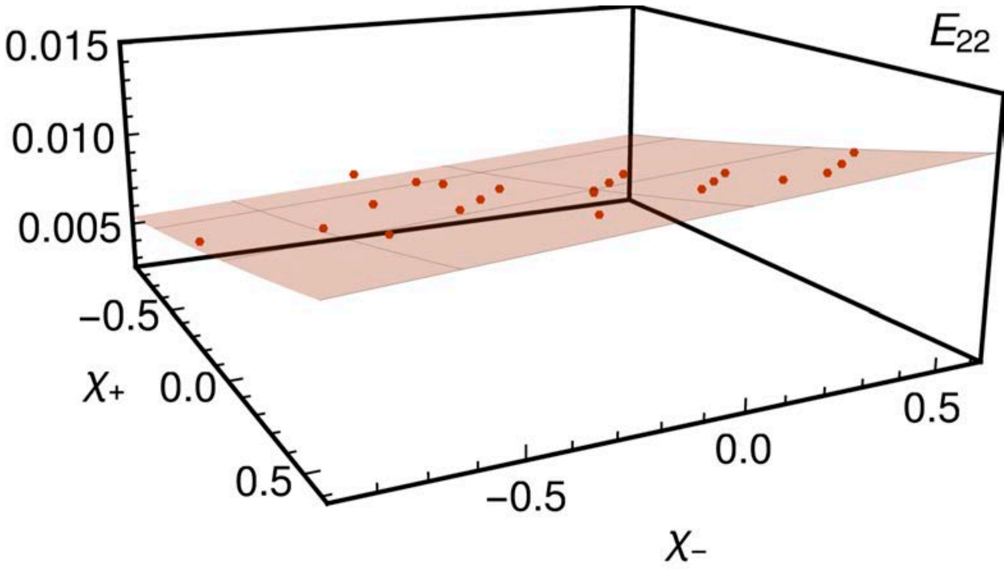
Earth vs. space-based: ringdown detections and black hole spectroscopy



[EB+, 1605.09286]

Multi-mode detectability: mass ratio and spin dependence



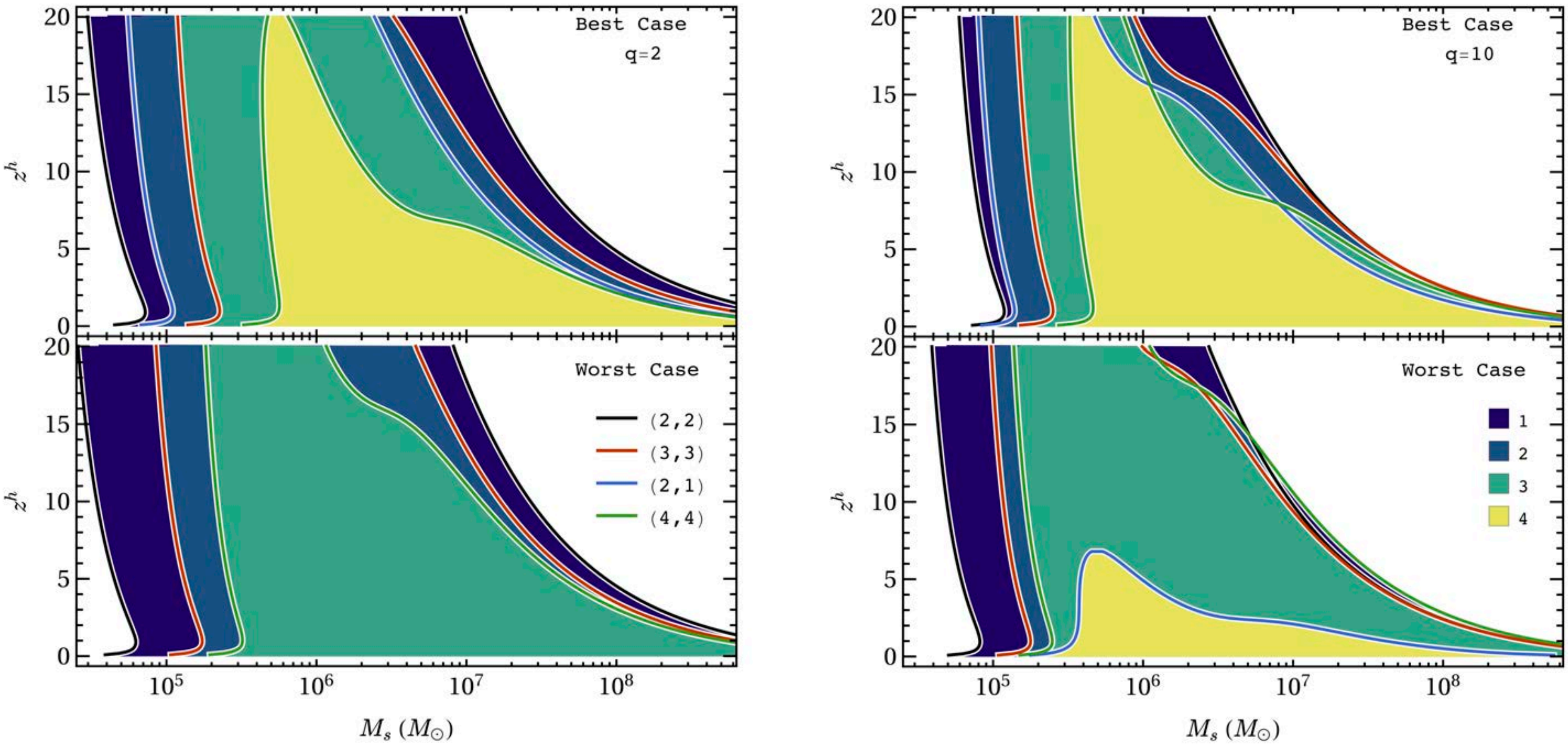


$$\chi_\pm \equiv \frac{m_1\chi_1 \pm m_2\chi_2}{m_1+m_2}$$

Strongest spin dependence: $\ell = 2, m = 1$

[Baibhav+, 1710.02156]

How many modes? Depends on spin. Best/worst case scenarios in LISA



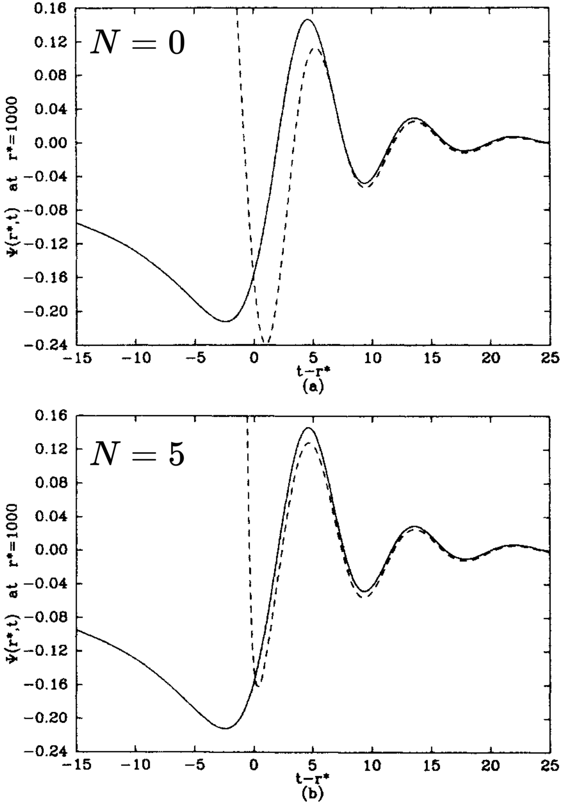
[Baibhav+EB, 1809.03500]

WITH GREAT SNR
COMES GREAT
RESPONSIBILITY.

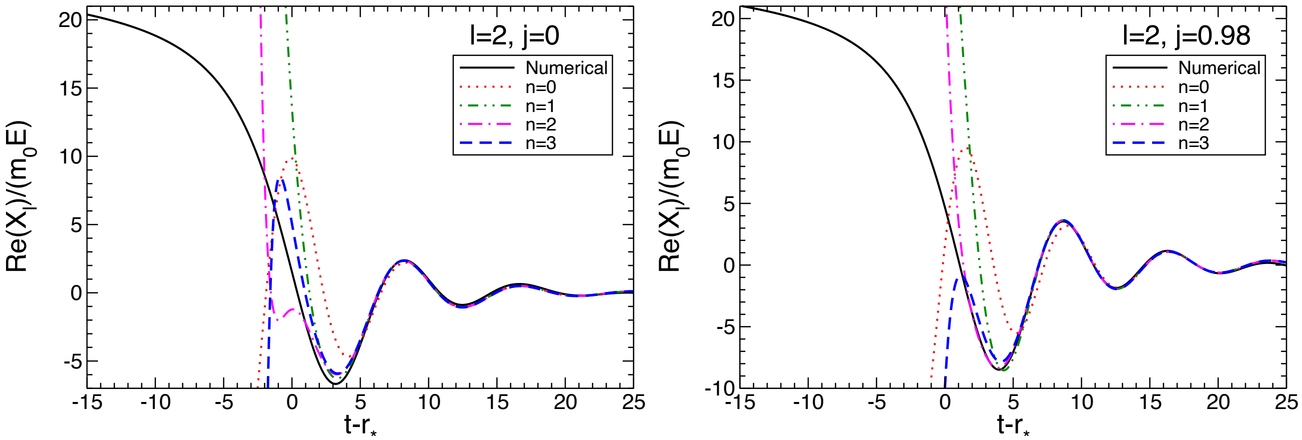
SPIDERMAN

Including overtones is crucial, even in linear perturbation theory

Leaver (1986): Green's function in Schwarzschild. Overtones: agreement well before peak
Zhang+: extension to Kerr (here for an ultrarelativistic infall along the z axis)



[Leaver, PRD, 1986]

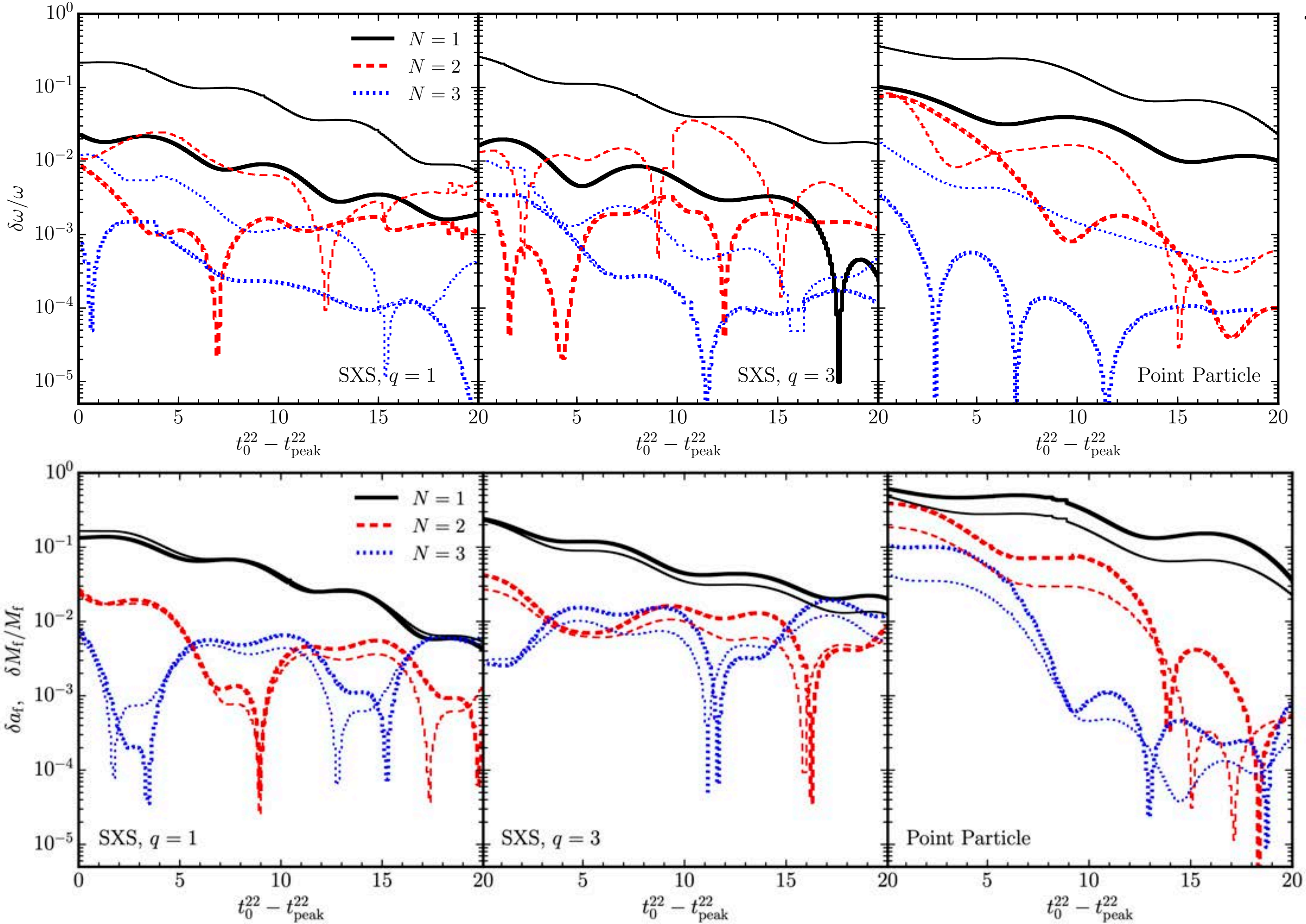


[Zhang+, 1710.02156]

"Excitation factors" in Kerr known
"Excitation coefficients" depend on initial data:
difficult, **unsolved** problem for comparable-mass mergers

[EB+Cardoso, gr-qc/0605118]

Systematic errors on QNM frequencies/mass+spin from SXS/PP waveforms



Top:
real part (thick)
imaginary part (thin)

1% determination of ω_{220} needs one overtone (better if two or three)

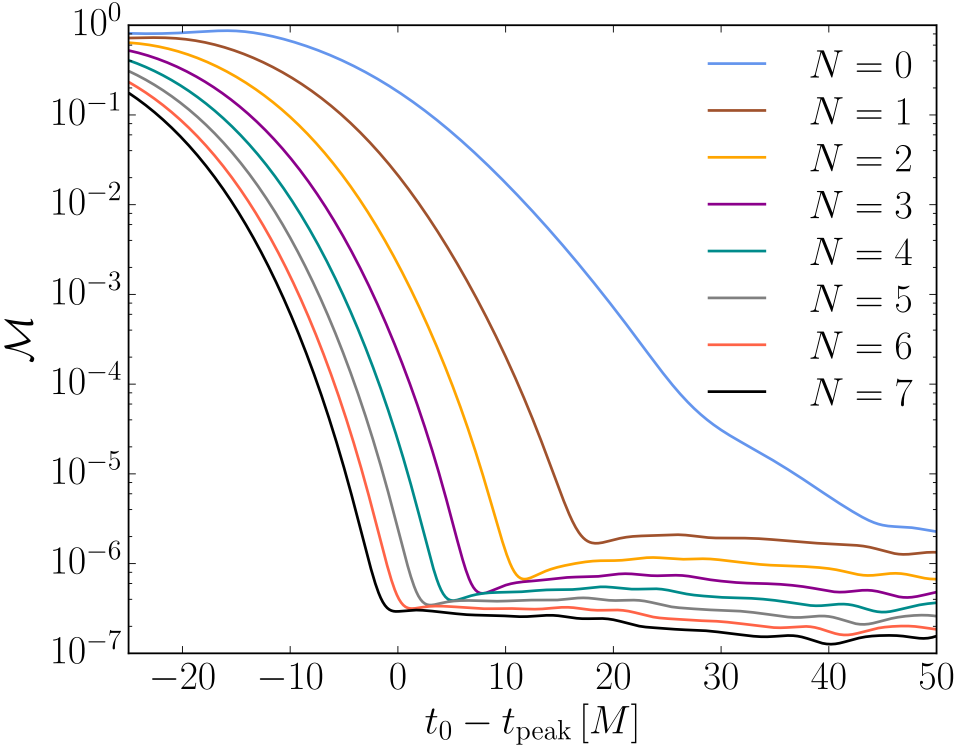
Bottom:
spin (thick)
mass (thin)

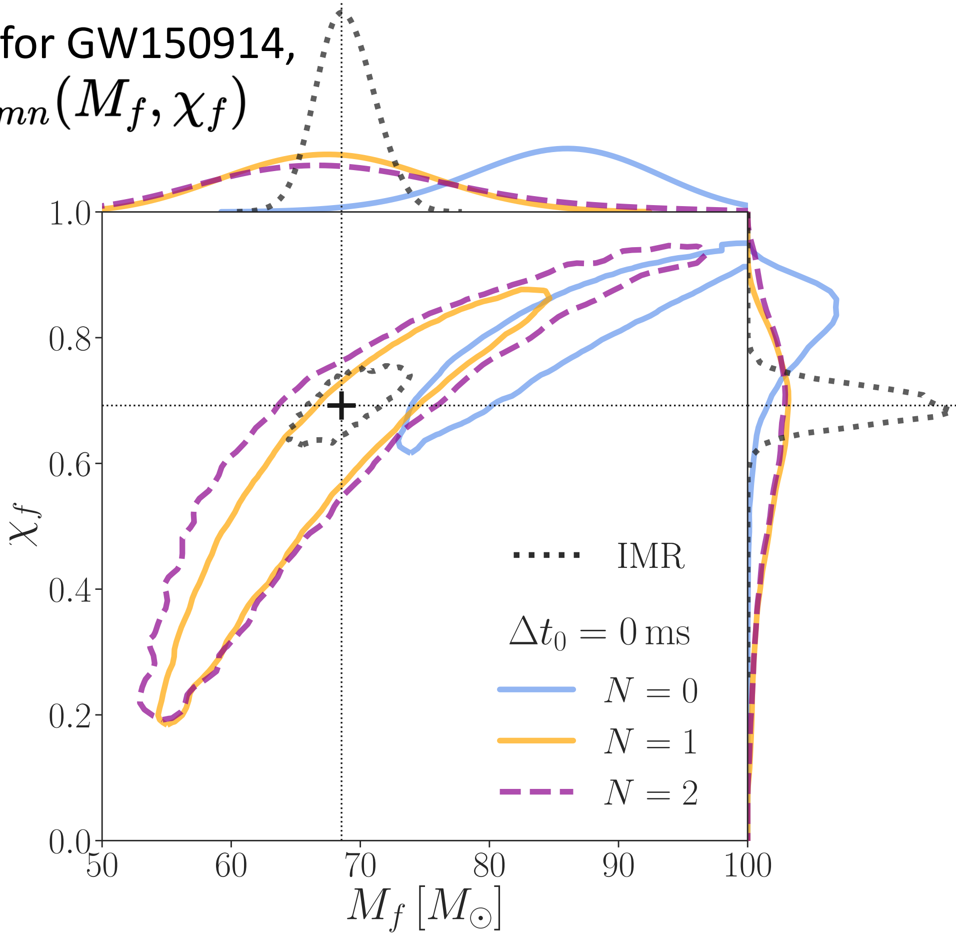
1% determination of mass and spin needs at least two modes

[Baibhav+, 1710.02156]

Systematic errors on mass and spin from fitting SXS waveforms

Overtones improve quality of consistency tests for GW150914, not a "genuine" spectroscopy test: $\omega_{\ell mn} = \omega_{\ell mn}(M_f, \chi_f)$





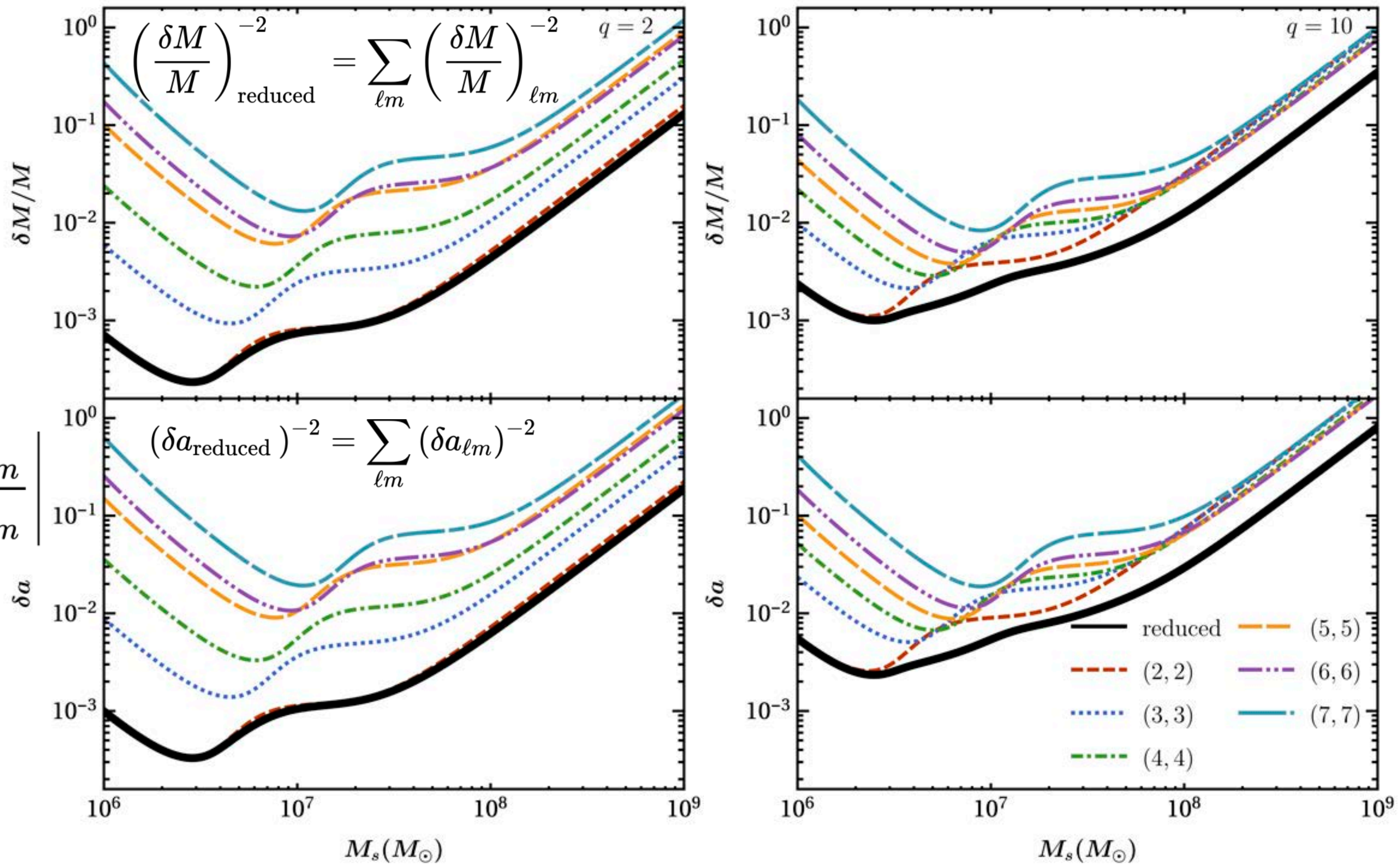
[Giesler+, 1903.08284; Isi+, 1905.00869]

Mass and spin measurement with multiple modes

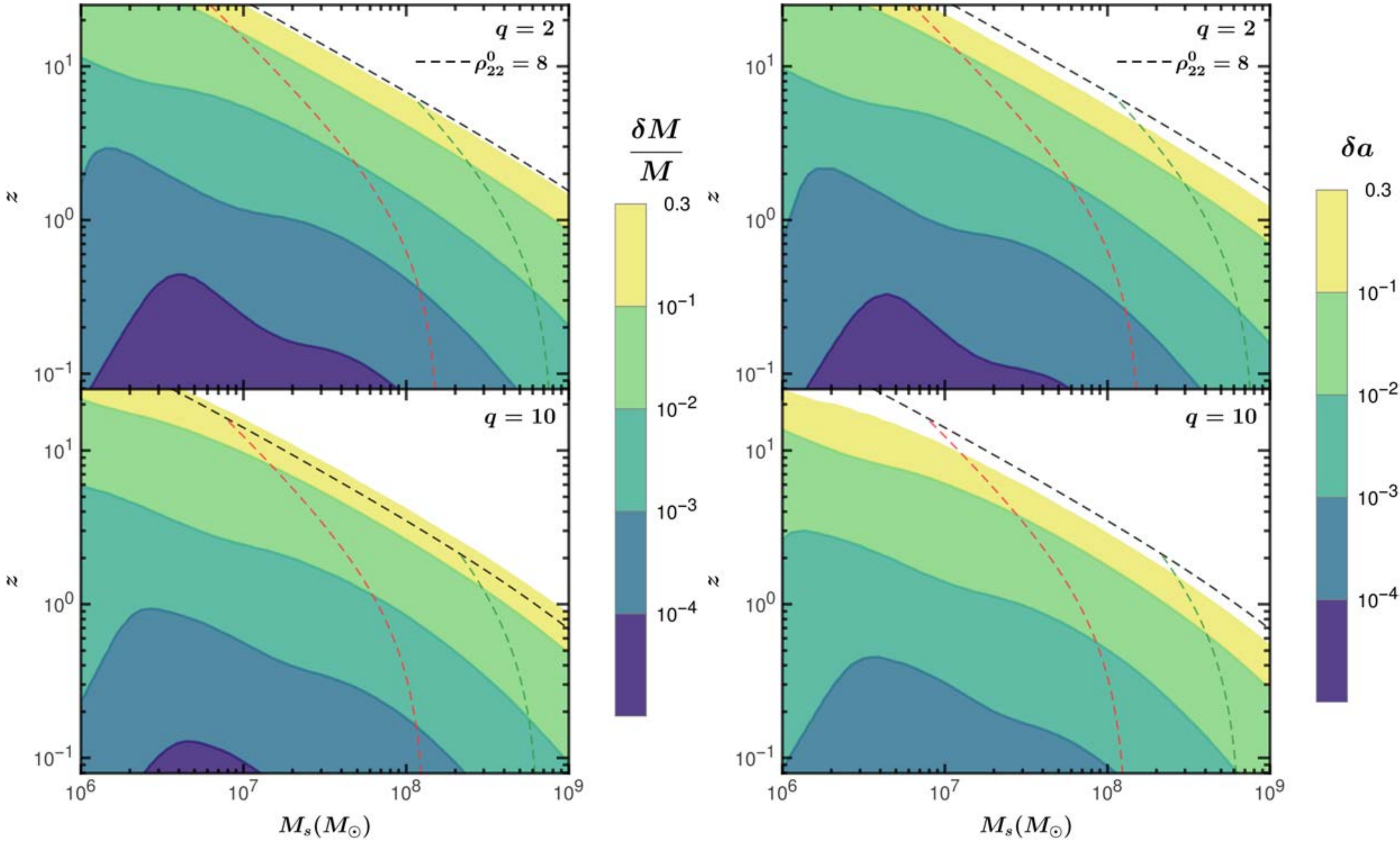
$$Q_{\ell m} = \pi f_{\ell m} \tau_{\ell m}$$

$$\delta a = \frac{1}{\rho_{\ell m}} \left| 2 \frac{Q_{\ell m}}{Q'_{\ell m}} \right|$$

$$\frac{\delta M}{M} = \frac{1}{\rho_{\ell m}} \left| 2 \frac{Q_{\ell m} f'_{\ell m}}{Q'_{\ell m} f_{\ell m}} \right|$$



Median errors on mass and spin combining multiple modes



Sky localization and distance determination

$$h_{\ell m}^{i}(t) = \mathcal{A}_{\ell m}^{i} e^{-(t-t_0)/\tau_{\ell m}} \cos(2\pi f_{\ell m} t + \Phi_{\ell m}^{i}) \qquad \Omega_{\ell m}^{i} \equiv \sqrt{\left(F_{+}^{i} Y_{+}^{\ell m}\right)^2 + \left(F_{\times}^{i} Y_{\times}^{\ell m}\right)^2}$$

Amplitudes:

$$\mathcal{A}_{\ell m}^{i} = \frac{M\Omega_{\ell m}^{i}}{d_L(z)} A_{\ell m}(q, \chi_1, \chi_2)$$

Amplitude ratio:

$$Q_A^{\ell m} = \left(\frac{\mathcal{A}_{\ell m}^{\mathrm{I}}}{\mathcal{A}_{\ell m}^{\mathrm{II}}}\right)^2$$

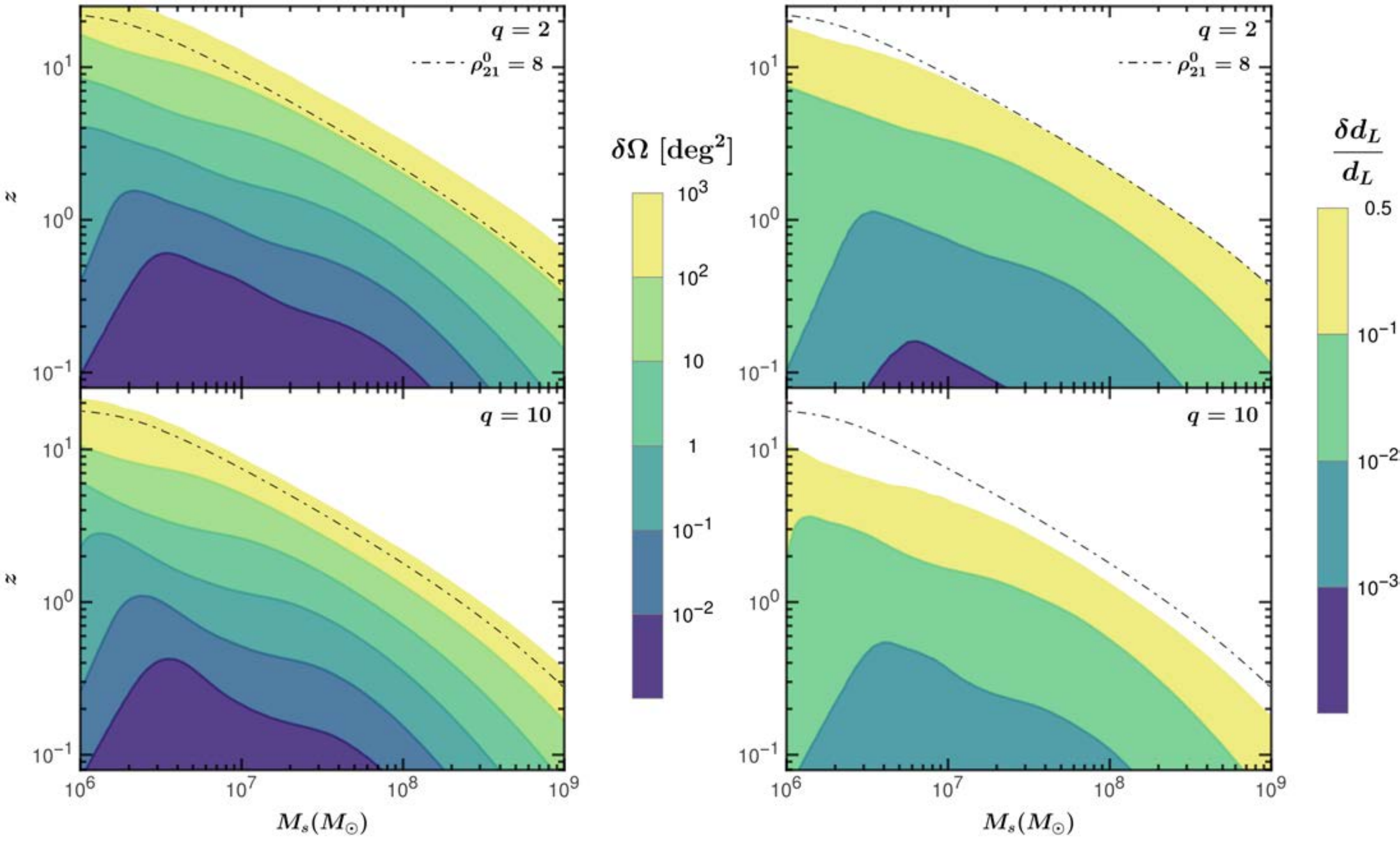
Phase difference:

$$\tan^{-1} Q_{\Phi}^{\ell m} = \Phi_{\ell m}^{\mathrm{II}} - \Phi_{\ell m}^{\mathrm{I}}$$

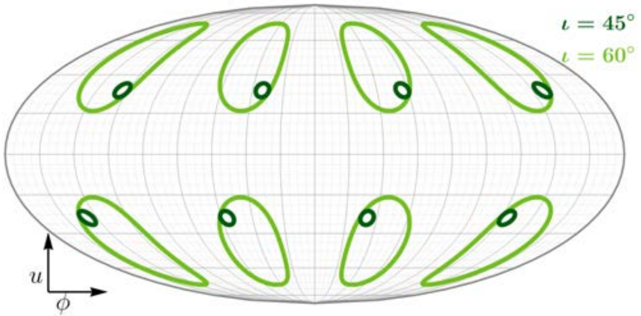
Relative antenna and polarization power:

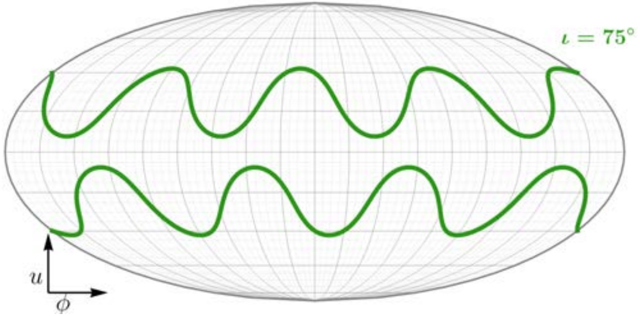
$$Q_1 = \frac{\left(F_{+}^{\mathrm{I}}\right)^2 + \left(F_{\times}^{\mathrm{I}}\right)^2}{\left(F_{+}^{\mathrm{II}}\right)^2 + \left(F_{\times}^{\mathrm{II}}\right)^2}$$

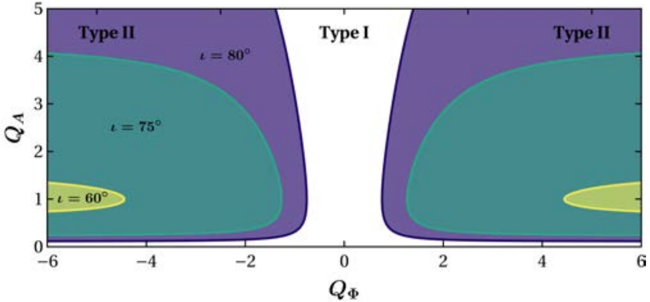
$$Q_2 = \frac{\left(F_{+}^{\mathrm{I}}\right)^2 + \left(F_{+}^{\mathrm{II}}\right)^2}{\left(F_{\times}^{\mathrm{I}}\right)^2 + \left(F_{\times}^{\mathrm{II}}\right)^2}$$

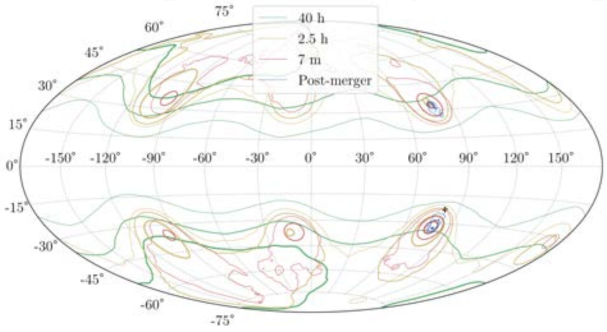


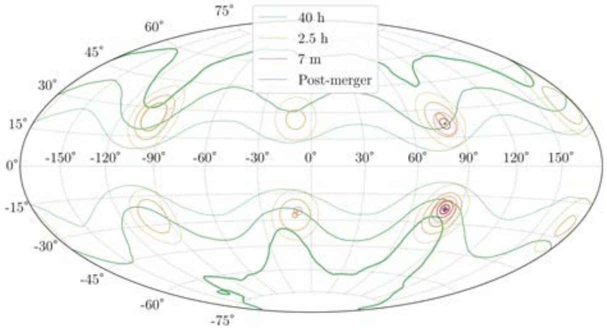
Sky localization: the eightfold way and higher harmonics











Assume inclination is known. Main observables:

$$Q^{22}_{\Phi}(\theta, \phi, \psi), Q^{22}_{A}(\theta, \phi, \psi)$$

Ignoring errors, these give contours of constant (θ, ϕ)

Degenerate positions witout orbital modulation and higher harmonics – but can do do better with better waveform models/PE

[Baibhav+, 2001.10011]
[Marsat+, 2003.00357]

Beyond GR: specific theories

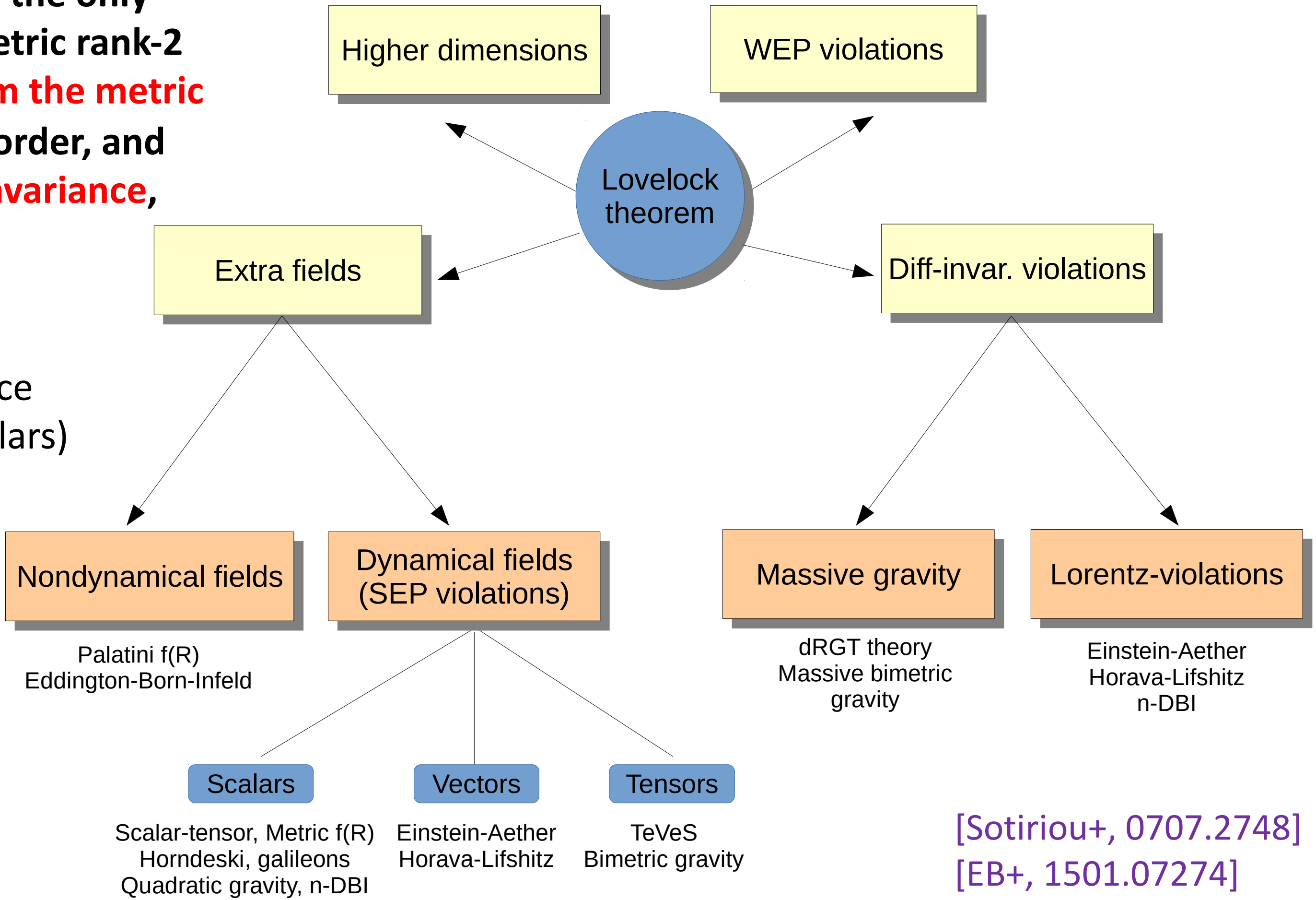
A guiding principle to modified GR: Lovelock's theorem

In four spacetime dimensions the only divergence-free (WEP) symmetric rank-2 tensor constructed solely from the metric and its derivatives up to 2nd order, and preserving diffeomorphism invariance, is the Einstein tensor plus Λ.

Generic modifications introduce additional fields (simplest: scalars)

Minimal requirements:

- Action principle
- Well-posed
- Testable predictions
- Black holes, neutron stars
- Cosmologically viable



[Sotiriou+, 0707.2748]
[EB+, 1501.07274]

Dynamical no-hair results in scalar-tensor theories

$$S = \frac{1}{16\pi}\int \sqrt{-g}d^4x\left[\phi R - \frac{\omega(\phi)}{\phi}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + M(\phi)\right] + \int \mathcal{L}_{\rm M}(g^{\mu\nu},\Psi)d^4x$$

Orbital period derivative: $\frac{\dot{P}}{P} = -\frac{\mu m}{r^3}\kappa_D(s_1 - s_2)^2 - \frac{8}{5}\frac{\mu m^2}{r^4}\kappa_1$

$$\kappa_D = 2\mathcal{G}\xi\left(\frac{\omega^2 - m_s^2}{\omega^2}\right)^{\frac{3}{2}}\Theta(\omega - m_s)$$

$$\kappa_1 = \mathcal{G}^2\left[12 - 6\xi + \xi\Gamma^2\left(\frac{4\omega^2 - m_s^2}{4\omega^2}\right)^{\frac{5}{2}}\Theta(2\omega - m_s)\right]$$

$$\xi = \frac{1}{2+\omega_{\rm BD}}$$

$$G = 1 - \xi(s_1 + s_2 - 2s_1 s_2)$$

$$\Gamma = 1 - 2\frac{s_1 m_2 + m_1 s_2}{m}$$

For black hole binaries, $s_1 = s_2 = \frac{1}{2}$ and dipole vanishes identically

Quadrupole: $\Gamma = 0$

Result extended to higher PN orders, BH-NS, and is exact in the large mass ratio limit [Will & Zaglauer 1989; Alsing+, 1112.4903; Mirshekari & Will, 1301.4680; Yunes+, 1112.3351; Bernard 1802.10201, 1812.04169, 1906.10735]

Ways around: matter (but EOS degeneracy), cosmological BCs (but small corrections), or **curvature itself sourcing the scalar field: dCS, EsGB** [Yagi+ 1510.02152]

The EFT viewpoint

Expand all operators in the action in terms of some length scale (must be macroscopic to be relevant for GW tests).

Theories: sum over curvature invariants with scalar-dependent coefficients

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R + \sum_{n=2}^{\infty} \ell^{2n-2}\mathcal{L}_{(n)}\right]$$

and more specifically, at order ℓ^4

EsGB dCS (dilaton+axion)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\Big\{R + \alpha_1\phi_1\ell^2 R_{\mathrm{GB}} + \alpha_2(\phi_2\cos\theta_m + \phi_1\sin\theta_m)\ell^2 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

$$+\lambda_{\mathrm{ev}}\ell^4 R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\delta\gamma}R_{\delta\gamma}^{\mu\nu} + \lambda_{\mathrm{odd}}\ell^4 R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\delta\gamma}\tilde{R}_{\delta\gamma}^{\mu\nu} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2\Big\}$$

Einsteinian cubic gravity (+parity-breaking) - causality constraints [Camanho+ 1407.5597]

Next order, no new DOFs [Endlich-Gorbenko-Huang-Senatore, 1704.01590]

$$S_{(4)} = \frac{\ell^6}{16\pi G}\int d^4x\sqrt{|g|}\left\{\epsilon_1\mathcal{C}^2 + \epsilon_2\tilde{\mathcal{C}}^2 + \epsilon_3\mathcal{C}\tilde{\mathcal{C}}\right\} \qquad \mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \quad \tilde{\mathcal{C}} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

[Cano-Ruipérez, 1901.01315; Cano-Fransen-Hertog, 2005.03671. See also work by Hui, Penco…]

Why Einstein-scalar-Gauss-Bonnet gravity? A loophole in no-hair theorems

Horndeski Lagrangian: most general scalar-tensor theory with second-order EOMs

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g}\mathcal{L}_i$$

$$G_i = G_i(\phi, X) \qquad \phi^2_{\mu\nu} \equiv \phi_{\mu\nu}\phi^{\mu\nu}$$

$$X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi \qquad \phi^3_{\mu\nu} \equiv \phi_{\mu\nu}\phi^{\nu\alpha}\phi^\mu_\alpha$$

$$\mathcal{L}_2 = G_2$$
$$\mathcal{L}_3 = -G_3\Box\phi$$
$$\mathcal{L}_4 = G_4R + G_{4X}\left[(\Box\phi)^2 - \phi^2_{\mu\nu}\right]$$
$$\mathcal{L}_5 = G_5G_{\mu\nu}\phi^{\mu\nu} - \tfrac{1}{6}G_{5X}\left[(\Box\phi)^3 + 2\phi^3_{\mu\nu} - 3\phi^2_{\mu\nu}\Box\phi\right]$$

Set:

$$G_2 = X + 8f^{(4)}X^2(3 - \ln X)$$
$$G_3 = 4f^{(3)}X(7 - 3\ln X)$$
$$G_4 = \tfrac{1}{2} + 4f^{(2)}X(2 - \ln X)$$
$$G_5 = -4f^{(1)}\ln X$$

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + X + f(\phi)\mathcal{G}\right)$$

$$\mathcal{G} \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Shift symmetry: invariance under $\phi \to \phi + c$, i.e. $G_i = G_i(X)$

EsGB is **a special case of Horndeski *and* of quadratic gravity**

[Kobayashi+, 1105.5723; Sotiriou+Zhou, 1312.3622; Maselli+, 1508.03044]

Scalar-tensor theory and spontaneous scalarization

- Action (in the Einstein frame):

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g^\star}\left[R^\star - 2g^{\star\mu\nu}\left(\partial_\mu\varphi\right)\left(\partial_\nu\varphi\right) - V(\varphi)\right] + S_M[\Psi, A^2(\varphi)g^\star_{\mu\nu}]$$

- Gravity-matter coupling:

$$\alpha(\varphi) \equiv d(\ln A(\varphi))/d\varphi$$

$$\alpha(\varphi) = \alpha_0 + \beta_0(\varphi - \varphi_0) + \dots$$

- Field equations:

$$G^\star_{\mu\nu} = 2\left(\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^\star_{\mu\nu}\partial_\sigma\varphi\partial^\sigma\varphi\right) - \frac{1}{2}g^\star_{\mu\nu}V(\varphi) + 8\pi T^\star_{\mu\nu}\,,$$

$$\Box_{g^\star}\varphi = -4\pi\alpha(\varphi)T^\star + \frac{1}{4}\frac{dV}{d\varphi}\,,$$

[Damour+Esposito-Farese, PRL 70, 2220 (1993); PRD 54, 1474 (1996)]

Scalarization threshold: a back-of-the-envelope argument

$$\Box_{g^*}\varphi = -4\pi\alpha(\varphi)T^* \qquad \alpha(\varphi) = \beta_0\varphi$$

$$-T^* = A^4(\epsilon^* - 3p^*) \sim \frac{3}{4\pi R^2}\frac{m}{R} \quad \text{for} \quad r < R$$

$$\nabla^2\varphi = \mathrm{sign}(\beta_0)\left[\frac{3|\beta_0|(m/R)}{R^2}\right]\varphi = \mathrm{sign}(\beta_0)\kappa^2\varphi$$

$$\beta_0 < 0 \Longrightarrow \varphi_{\mathrm{inside}} = \varphi_{\mathrm{c}}\frac{\sin(\kappa r)}{\kappa r}$$

$$\varphi_{\mathrm{c}} = \frac{\varphi_0}{\cos(\kappa R)} \gg \varphi_0 \qquad \boxed{\kappa R \sim \pi/2}$$

$$\boxed{m/R \sim 0.2 \Longrightarrow \beta \sim -4}$$

[Damour+Esposito-Farese, PRL 70, 2220 (1993)]

No-hair conditions in EsGB

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathscr{G}\right] + S_\mathrm{m}[g_{\mu\nu},\psi]$$

$$\mathscr{G} \equiv R_{\mu\nu\rho\sigma}R^{\mu\upsilon\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$\Box\varphi = -f_{,\varphi}\mathscr{G}$$

$$G_{\mu\nu} = T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}\partial_\rho\varphi\partial^\rho\varphi + \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi - g_{\gamma(\mu}g_{\nu)\delta}\epsilon^{\sigma\delta\rho\chi}\nabla_\tau[{}^*R^{\gamma\tau}{}_{\rho\chi}\nabla_\sigma f]$$

Matter: zero in vacuum

GB contribution $\propto f_{,\varphi}$

Kerr is a solution with constant scalar field if: $\boxed{f_{,\varphi}(\varphi_0) = 0}$

Dilatonic theories $f \sim \exp(\varphi)$ and shift-symmetric theories $f \sim \varphi$ **do not have a GR limit!**

No-hair theorem: in addition, $\boxed{f_{,\varphi\varphi}\mathscr{G} < 0}$

[Silva+, 1711.02080]

A proof, and a heuristic argument

$$\Box\varphi = -f_{,\varphi}\mathscr{G}$$

$$\int_{\mathscr{V}} \mathrm{d}^4x\sqrt{-g}\left[f_{,\varphi}\Box\varphi + f_{,\varphi}^2(\varphi)\mathscr{G}\right] = 0$$

Integrate by parts, divergence theorem:

$$\int_{\mathscr{V}} \mathrm{d}^4x\sqrt{-g}\left[f_{,\varphi\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - f_{,\varphi}^2(\varphi)\mathscr{G}\right] = \int_{\partial\mathscr{V}} \mathrm{d}^3x\sqrt{|h|}f_{,\varphi}n^\mu\nabla_\mu\varphi$$

The RHS vanishes for stationary, asymptotically flat spacetimes; if $\boxed{f_{,\varphi\varphi}\mathscr{G} < 0}$ both terms on the LHS vanish separately, i.e. $\varphi = \varphi_0 = c$

In alternative, linearize the scalar field equation: $[\Box + f_{,\varphi\varphi}(\varphi_0)\mathscr{G}]\delta\varphi = 0$

$m_{\mathrm{eff}}^2 = -f_{,\varphi\varphi}\mathscr{G}$ is an effective mass for the perturbation – tachyonic instability?

[Silva+, 1711.02080]

EsGB: black hole scalarization and other solutions

Einstein-dilaton-Gauss-Bonnet: $f = e^{\alpha\varphi}$

Kerr not a solution, minimum BH mass

[Mignemi-Stewart 93, Kanti+ 96, Pani-Cardoso 09, Yunes-Stein 11…]

Shift-symmetric Gauss-Bonnet: $f = a\varphi$

Kerr not a solution!

[Sotiriou-Zhou 14, Barausse-Yagi 15, Benkel+ 16…]

Minimal scalarization model: $f = \frac{1}{8}\eta\varphi^2$

[Silva+ 1711.02080]

Nonminimal scalarization model: $f = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$

[Doneva+ 1711.01187]

Polynomial, inverse polynomial, logarithmic…

[Antoniou+ 1711.03390/07431; Brihaye-Ducobu 1812.07438…]

Radial instability of the minimal scalarization model

Are the solutions stable under radial perturbations?

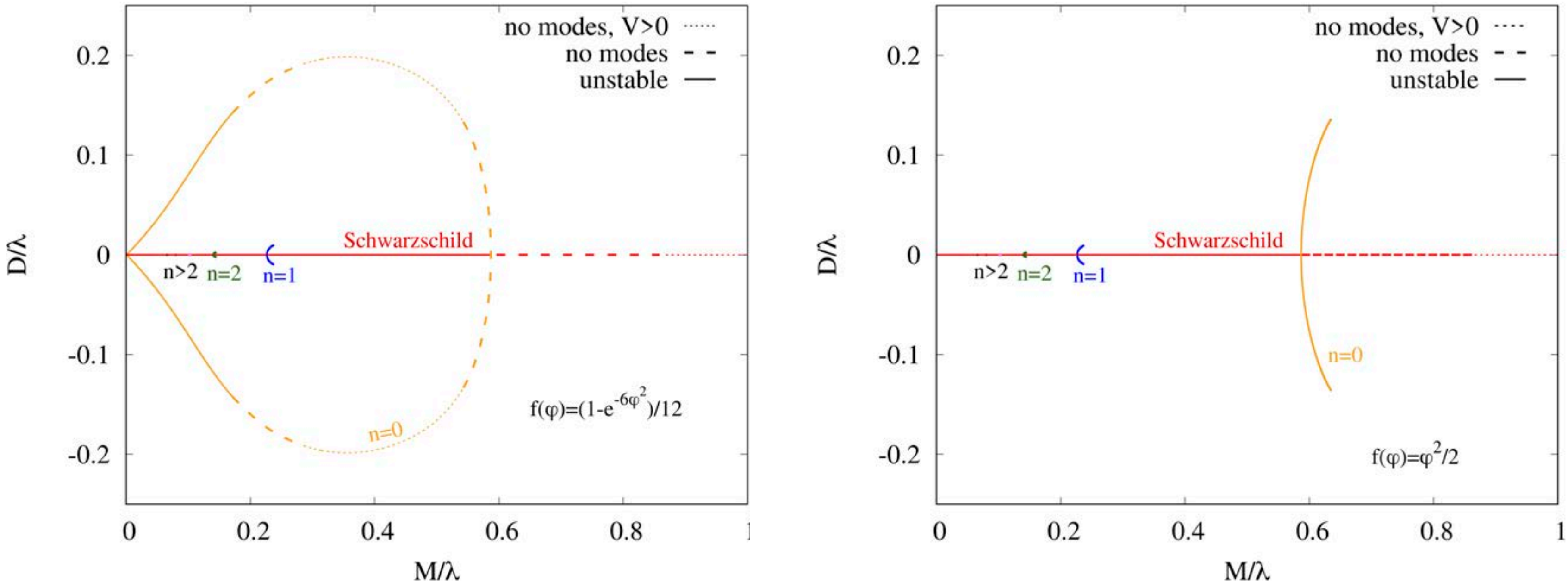
Dashed/dotted: stable (no unstable modes, or positive potential); solid: unstable

$M > M_{\rm thr} = 0.587\eta^{1/2}$: Schwarzschild is stable; threshold is coupling-independent

Intermediate mass: nodeless scalarized BHs with exponential coupling are stable

No BHs are stable below a certain mass (as in EdGB)

(scalar charge Q)

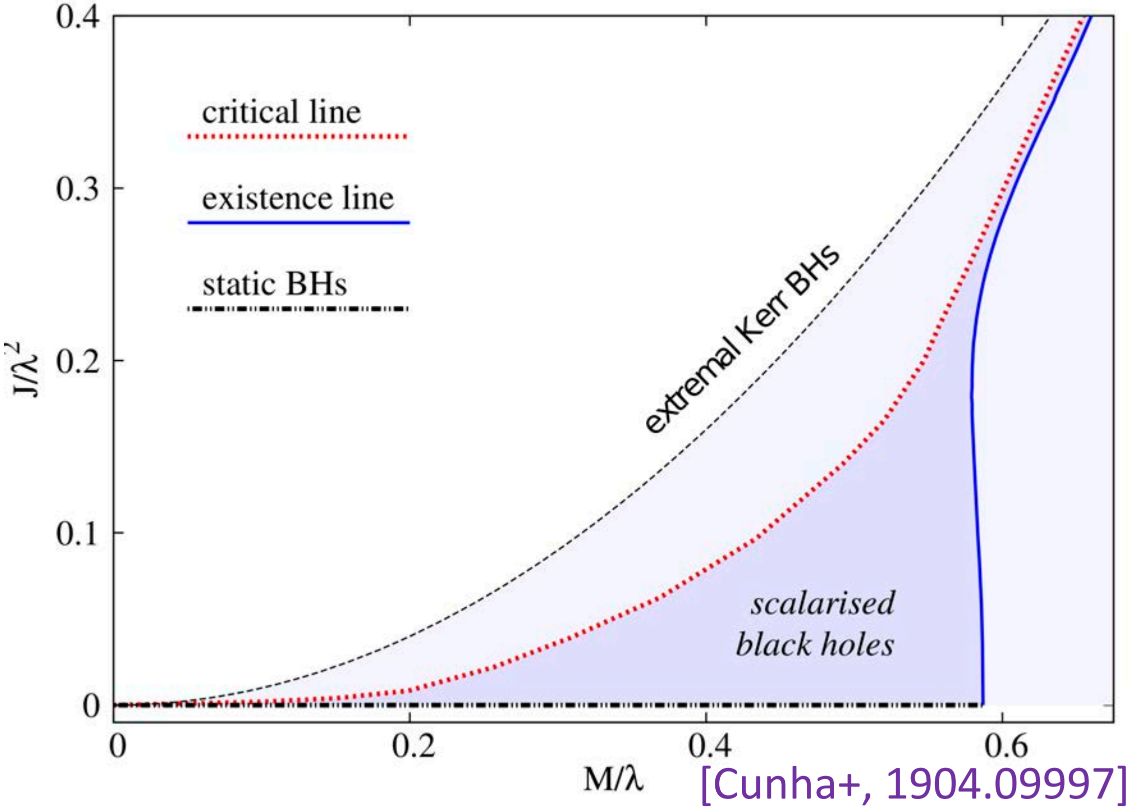


[Blazquez-Salcedo+, 1805.05755]

What about spin?

$$f = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$$

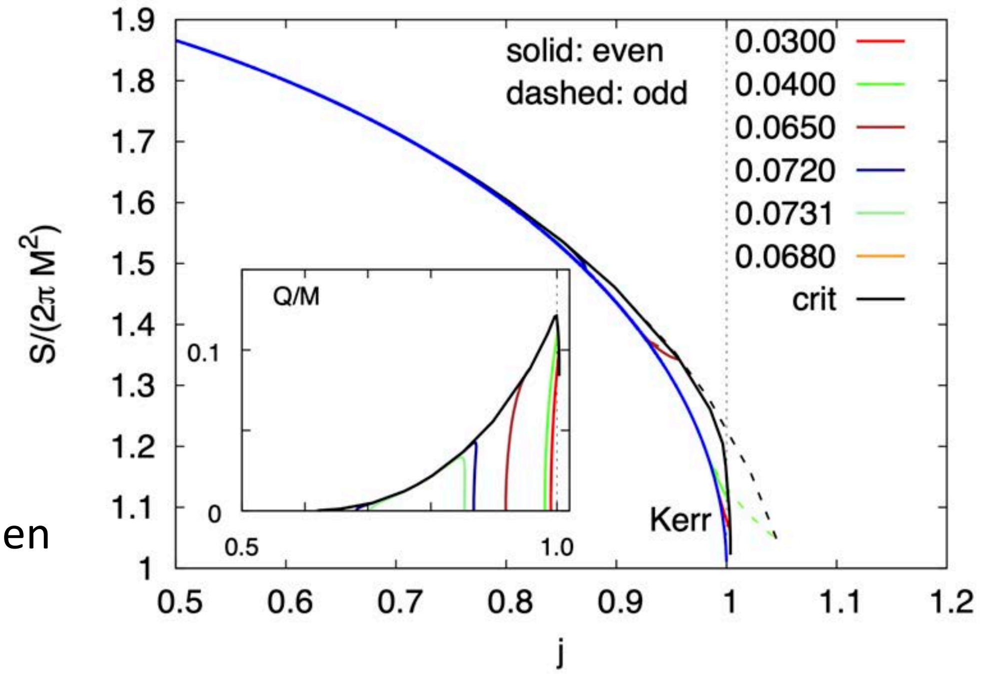
- For each spin J, scalarized BHs exist between two critical mass values (lowest bound: zero in static limit)
- Scalarized black holes are entropically favored
- Spin reduces difference between scalarized/unscalarized solutions: differences nearly unmeasurable for $j \geq 0.5$ (LIGO range!)
- Coupling dependence?



[Cunha+, 1904.09997]

Spin-induced scalarization

- Take a second look at $\Box\varphi = -f_{,\varphi}\mathscr{G}$
- Gauss-Bonnet invariant **changes sign** for $j \geq 0.5$! Spin-induced instability
- Even for quadratic coupling, spin-induced scalarized black holes are **entropically favored** over Kerr
- Small differences in charge, mass etc between quadratic and exponential coupling



- Quasinormal modes? May be easier via time evolutions
- Dynamical stability?

[Dima+, 2006.03095; Herdeiro+, 2009.03904; EB+, 2009.03905]

Black hole thermodynamics, skeletonization and the two-body Lagrangian

- Analytical solutions for generic coupling functions (up to fourth order in the EsGB coupling constant) satisfy the first law of black hole thermodynamics

- Skeletonization à la Eardley: slow inspiral means that black holes evolve with **constant Wald entropy** and a nontrivial asymptotic value of the scalar field

- Padé resummation seems to correctly predict **poles** in coupling of black holes to scalar fields

- EsGB-induced corrections to the (conservative) two-body Lagrangian:
  formally 1PN contribution, but for small coupling, effectively a 3PN term
  "Miraculously", no regularization is needed to solve the EOMs at 1PN: "simple" Fock integral

- d+1 formulation:
  nonlinear canonical momenta, multivalued ADM Hamiltonian, strong-field breakdown

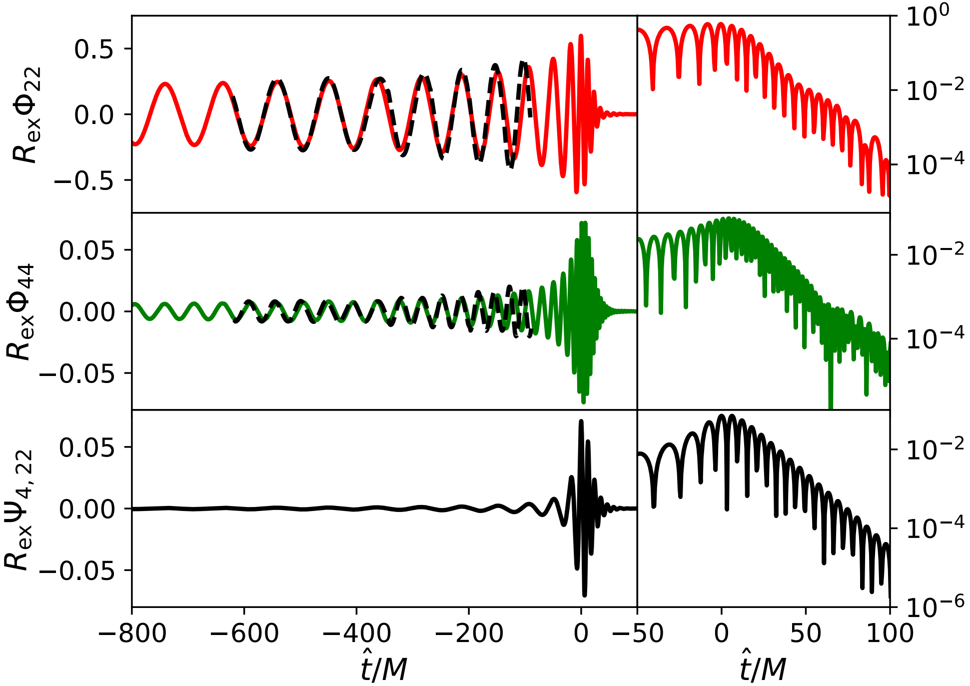
[Julié+EB 1909.05258, 2004.00003; Witek+ 2004.00009; Ripley+Pretorius]

Binaries in Einstein-scalar-Gauss-Bonnet

- Post-Newtonian calculations in the weak coupling limit:
  [Yagi+, 1110.5990]
  Higher-order in coupling, generic EsGB:
  [Julié+, 1909.05258]

- Dynamical scalarization:
  [Khalil+, 1906.08161]

- Numerical simulations (weak coupling limit):
  Scalar waveforms
  Scalar-led QNMs + gravitational-led QNMs
  [Witek+, 1810.05177]



- Related work in dynamical Chern-Simons (weak coupling limit):
  Scalar and gravitational waveforms
  [Okounkova+ 1705.07924, 1906.08789]

Well posedness

- Papallo-Reall: EsGB not strongly hyperbolic in the generalized harmonic gauge

  [Papallo-Reall, 1704.04730]

- Ripley-Pretorius: evidence for hair in spherical collapse in the weak-coupling limit, but well-posedness issues in spherical collapse in shift-symmetric EdGB

  "there are open sets of initial data for which the character of the system of equations changes from hyperbolic to elliptic type in a compact region of the spacetime […]
  it is conceivable that a well-posed formulation of EdGB gravity (at least within spherical symmetry) may be possible if the equations are appropriately treated as mixed-type"

  [Ripley-Pretorius, 1902.01468]
  [Ripley-Pretorius, 1903.07543]

- Does Horndeski generally lead to caustics? Do global solutions exist?

  [Babichev 1602.00735]
  [Bernard-Lehner-Luna, 1904.12866]

Time evolution of the full Einstein-scalar-Gauss-Bonnet theory?

- Built action generalizing Gibbons-Hawking-York and Myers to ESGB theories
- ADM Lagrangian and Hamiltonian and d+1 decomposition generalized to EsGB
- Canonical momenta of ESGB theories are nonlinear in the extrinsic curvature: "accelerations" are functions of metric, scalar field and their first derivatives

$$\frac{\partial \pi_{ij}}{\partial K_{ab}} \mathcal{A}_{ab} + \frac{\partial \pi_{ij}}{\partial K_{\varphi}} \mathcal{A}_{\varphi} = \mathcal{F}_{ij}$$

$$\frac{\partial \pi_{\varphi}}{\partial K_{ab}} \mathcal{A}_{ab} + \frac{\partial \pi_{\varphi}}{\partial K_{\varphi}} \mathcal{A}_{\varphi} = \mathcal{F}_{\varphi}$$

- Implications:
    1) the ADM Hamiltonian is generically multivalued, and the associated Hamiltonian evolution is not predictable
    2) the d+1 equations of motion are quasilinear and may break down in strongly curved, highly dynamical regimes
- Ripley-Pretorius: evidence for hair in spherical collapse in the weak-coupling limit, but well-posedness issues in spherical collapse in shift-symmetric EdGB

[Julié+EB, 2004.00003; see also Witek+, 2004.00009]

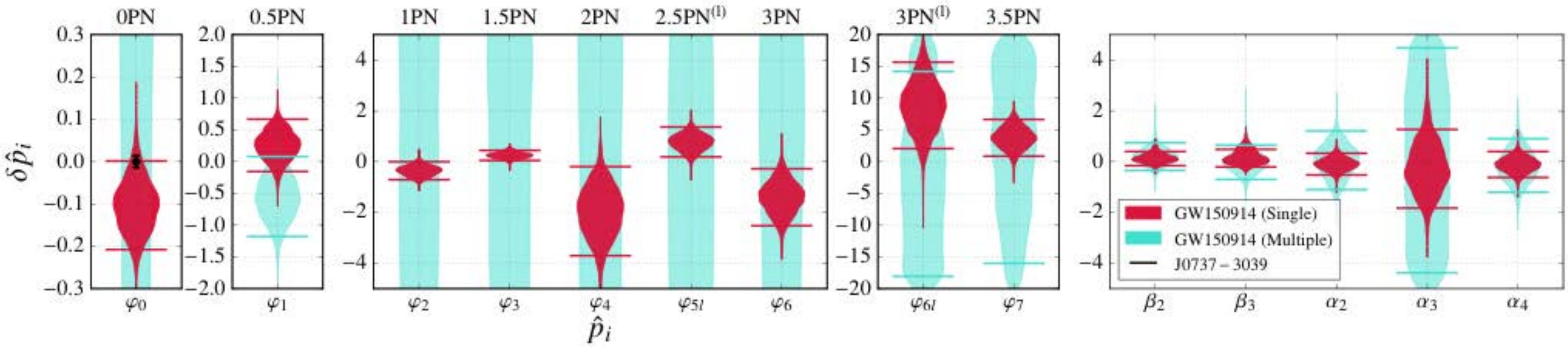
Specific Theories Summary

- EsGB: subclass of Horndeski theory that evades no-hair theorems
- Scalarized solution exist, are radially stable (as long as backreaction is included), can differ sensibly from GR
- Stable, nonspinning scalarized solutions are well motivated in EFT
- Scalarized solution become close to GR for spins of interest to LIGO remnant - more interesting phenomenology for spin-induced scalarization
- BHBs produce dipolar radiation [Yagi+ 1510.02152; Julié+, 1909.05258]
- Binaries have been simulated in the weak-coupling limit [Witek+ 1810.05177]
- Full merger? Open issues with well posedness in the strong-coupling limit [Papallo-Reall, Ripley-Pretorius, Bernard+, Julié+EB…]

Beyond GR: parametrization

Inspiral: GR solution known, parametrized post-Einstein

$$\tilde{h}(f) = \tilde{A}_{\rm GR}(f)\left[1 + \alpha_{\rm ppE}\, v(f)^a\right] e^{i\Psi_{\rm GR}(f) + i\beta_{\rm ppE}\, v(f)^b}$$



[Yunes-Pretorius+, 0909.3328]

Mapping to theories – can we do the same for ringdown?

Table 2 Mapping of ppE parameters to those in each theory for a black hole binary

Theory	$\beta_{\rm ppE}$	b
Scalar–tensor [36, 179, 180]	$-\frac{5}{1792}\dot{\phi}^2\eta^{2/5}\left(m_1 s_1^{\rm ST} - m_2 s_2^{\rm ST}\right)^2$	-7
EdGB, D^{2}GB [23]	$-\frac{5}{7168}\zeta_{\rm GB}\frac{\left(m_1^2 s_2^{\rm GB} - m_2^2 s_1^{\rm GB}\right)^2}{m^4\eta^{18/5}}$	-7
dCS [181]	$\frac{1549225}{11812864}\frac{\zeta_{\rm CS}}{\eta^{14/5}}\left[\left(1-\frac{231808}{61969}\eta\right)\chi_s^2 + \left(1-\frac{16068}{61969}\eta\right)\chi_a^2 - 2\delta_m\chi_s\chi_a\right]$	-1
EA [182]	$-\frac{3}{128}\left[\left(1-\frac{c_{14}}{2}\right)\left(\frac{1}{w_2^{\rm Æ}} + \frac{2c_{14}c_+^2}{(c_+ + c_- - c_- c_+)^2 w_1^{\rm Æ}} + \frac{3c_{14}}{2w_0^{\rm Æ}(2-c_{14})}\right) - 1\right]$	-5
Khronometric [182]	$-\frac{3}{128}\left[(1-\beta_{\rm KG})\left(\frac{1}{w_2^{\rm KG}}\frac{3\beta_{\rm KG}}{2w_0^{\rm KG}(1-\beta_{\rm KG})}\right) - 1\right]$	-5
Extra dimension [183]	$\frac{25}{851968}\left(\frac{dm}{dt}\right)\frac{3-26\eta+34\eta^2}{\eta^{2/5}(1-2\eta)}$	-13
Varying G [151]	$-\frac{25}{65536}\dot{G}\mathcal{M}$	-13
Mod. disp. rel. [184]	$\frac{\pi^{2-\alpha_{\rm MDR}}}{(1-\alpha_{\rm MDR})}\frac{D_{\alpha_{\rm MDR}}}{\lambda_{\mathbb{A}}^{2-\alpha_{\rm MDR}}}\frac{\mathcal{M}^{1-\alpha_{\rm MDR}}}{(1+z)^{1-\alpha_{\rm MDR}}}$	$3(\alpha_{\rm MDR}-1)$

Scalar, electromagnetic and gravitational perturbations in GR

Gravitational perturbations of a Schwarzschild BH: Regge-Wheeler/Zerilli equations

$$f\frac{d}{dr}\left(f\frac{d\Phi}{dr}\right)+\left[\omega^2-fV_\pm\right]\Phi=0 \qquad f=1-\frac{r_H}{r}$$

Isospectrality: the odd/even potentials

$$V_-=\frac{\ell(\ell+1)}{r^2}-\frac{3r_H}{r^3}$$

$$V_+=\frac{9\lambda r_H^2 r+3\lambda^2 r_H r^2+\lambda^2(\lambda+2)r^3+9r_H^3}{r^3(\lambda r+3r_H)^2}$$

have the same quasinormal mode spectrum [Chandrasekhar-Detweiler 1975]

Scalar, electromagnetic and (odd) gravitational perturbations:

$$V_s=\frac{\ell(\ell+1)}{r^2}+(1-s^2)\frac{r_H}{r^3}$$

[e.g. EB+, 0905.2975]

Generic (but decoupled) corrections to GR potentials

Modifications to the gravity sector and/or beyond Standard Model physics: expect

- small modifications to the functional form of the potentials – parametrize!
- coupling between the wave equations (more later)

$$V = V_\pm + \delta V_\pm \qquad \delta V_\pm = \frac{1}{r_H^2} \sum_{j=0}^{\infty} \alpha_j^\pm \left(\frac{r_H}{r}\right)^j \qquad \omega_{\mathrm{QNM}}^\pm = \omega_0^\pm + \sum_{j=0}^{\infty} \alpha_j^\pm e_j^\pm$$

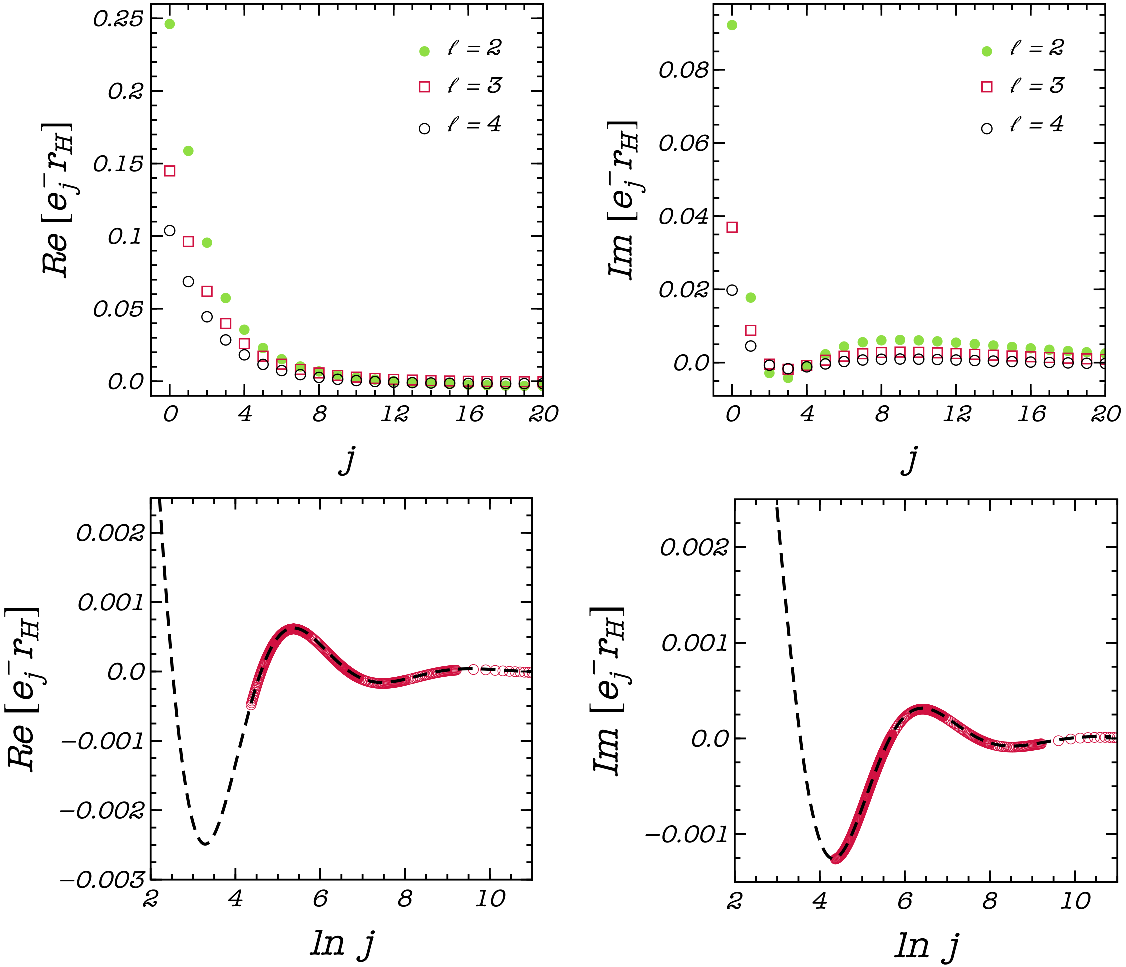
$$V = V_s + \delta V_s \qquad \delta V_s = \frac{1}{r_H^2} \sum_{j=0}^{\infty} \beta_j^s \left(\frac{r_H}{r}\right)^j \qquad \omega_{\mathrm{QNM}}^s = \omega_0^s + \sum_{j=0}^{\infty} \beta_j^s d_j^s$$

Maximum of $f(r)\alpha_j^\pm \left(\frac{r_H}{r}\right)^j$ is $\alpha_j^\pm \frac{(1+1/j)^{-j}}{j+1}$, so corrections are small if:

$$(\alpha_j^\pm, \beta_j^s) \ll (1+1/j)^j (j+1)$$

[Cardoso+, 1901.01265]

Correction coefficients and their asymptotic behavior



QNM frequency correction coefficients by direct integration [Pani, 1305.6759]

Asymptotics:

$$e_j \propto \frac{j^{2i\omega_0 r_H}}{j} = \frac{e^{2ir_H\omega_R \ln j}}{j^{1+2r_H\omega_I}}$$

Damped oscillatory behavior for large j

Fitting the numerics by

$$e_j \sim \frac{\kappa}{j^\beta}\sin(\gamma \ln j + \zeta)$$

confirms this.

[Pani, 1305.6759]

Generic isospectrality breaking

Isospectrality follows from the existence of a "superpotential" such that:

$$fV_\pm = W_0^2 \mp f\frac{dW_0}{dr} - \frac{\lambda^2(\lambda+2)^2}{36r_H^2} \qquad W_0 = \frac{3r_H(r_H - r)}{r^2(3r_H + \lambda r)} - \frac{\lambda(\lambda+2)}{6r_H}$$

Perturb to find conditions for isospectrality to hold:

$$2\frac{d\delta W}{dr} = \delta V_- - \delta V_+ \qquad 4\frac{W_0}{f}\delta W = \delta V_+ + \delta V_-$$

Preserving isospectrality needs fine tuning!

$$\alpha_0^+ = \alpha_0^-$$
$$\alpha_1^+ = \alpha_1^-$$
$$\alpha_2^+ = \alpha_2^- + \frac{6\left(\alpha_0^- - \alpha_1^-\right)}{\lambda(\lambda+2)}$$

[Chandrasekhar-Detweiler 1975]

Example 1: EFT

EFT corrections quartic in the curvature lead to a modified Regge-Wheeler equation:

$$\frac{d^2\Psi_-}{dr_\star^2} + \left[\omega^2 - f(V_- + \delta V_-)\right]\Psi_- = 0$$

$$\delta V_- = \epsilon_2 \frac{18(\ell+2)(\ell+1)(\ell-1)r_H^8}{r^{10}}$$

Trivially read off the correction coefficient: $\alpha_{10}^{-} = 18(\ell+2)(\ell+1)(\ell-1)\epsilon_2$

Plug into $\omega_{\mathrm{QNM}}^{\pm} = \omega_0^{\pm} + \sum_{j=0}^{\infty} \alpha_j^{\pm} e_j^{\pm}$

to find

$$r_H\omega = r_H\omega_0 + (0.0663354 + 0.117439\mathrm{i})\epsilon_2(\ell-1)(\ell+1)(\ell+2)$$

in agreement with numerical integrations.

[Cardoso+, 1808.08962]

Example 2: Reissner-Nordström

Odd gravitational perturbations of Reissner-Nordström satisfy

$$f_{\rm RN}\frac{d}{dr}\left(f_{\rm RN}\frac{d\Phi}{dr}\right)+(\omega^2-f_{\rm RN}V_{\rm RN})\Phi=0 \qquad f_{\rm RN}=1-\frac{2M}{r}+\frac{Q^2}{r^2}$$

A simple change of variables brings the wave equation in our "canonical" form, with

$$V_{\rm RN}=\frac{\ell(\ell+1)}{r^2}+\frac{4r_Hr_-}{r^4}-\frac{3(r_H+r_-)}{2r^3}-\frac{\left[4(\ell-1)(\ell+2)r_Hr_-+\frac{9}{4}(r_H+r_-)^2\right]^{1/2}}{r^3}$$

for small charge.

Read off coefficients to find:

$$\begin{aligned}\omega_{\rm QNM}&=\left(1-\frac{r_-}{r_H}\right)\left(\frac{2\Omega_0}{r_H}+e_0\alpha_0^-+e_3\alpha_3^-+e_4\alpha_4^-\right)\\&=\frac{\Omega_0}{M}+\frac{(0.0258177-0.002824i)Q^2}{M^3}\end{aligned}$$

TABLE II. Relative percentage errors on the real and imaginary parts of the QNMs for RN BHs, as a function of the charge-to-mass ratio Q/M.

Q/M	Δ_R	Δ_I
0.00	0%	0%
0.05	0.11%	0.042%
0.10	0.43%	0.17%
0.20	1.7%	0.66%
0.30	3.8%	1.5%
0.40	6.8%	2.6%
0.50	11%	4.2%

Example 3: Klein-Gordon in slowly rotating Kerr

At linear order in the spin parameter:

$$f\frac{d}{dr}\left(f\frac{d}{dr}\right)\Phi + \left(\omega^2 - fV_0 - \frac{4amM\omega}{r^3}\right)\Phi = 0$$

i.e.

$$f\frac{d}{dr}\left(f\frac{d}{dr}\right)\Phi + \left[\left(\omega - \frac{am}{r_H^2}\right)^2 - f\left(V_0 - \frac{2am\omega}{r_H^2} - \frac{2am\omega}{r_H^2}\frac{r_H}{r} - \frac{2am\omega}{r_H^2}\left(\frac{r_H}{r}\right)^2\right)\right]\Phi = 0$$

Correction coefficients to the scalar wave equation:

$$\beta_0^0 = \beta_1^0 = \beta_2^0 = -2am\omega_0^0$$

$$\omega_{\mathrm{QNM}} = \omega_0^0 + \frac{am}{r_H^2} - 2am\omega_0^0\left(d_0^0 + d_1^0 + d_2^0\right)$$

TABLE III. Relative percentage errors in the real and imaginary parts of the QNM frequencies for scalar perturbations around a slowly spinning black hole, as a function of the BH angular momentum a/M.

a/M	Δ_R	Δ_I
0	0%	0%
10^{-4}	0.0050%	0.83%
10^{-3}	0.049%	5.1%
10^{-2}	0.49%	34%

Coupled perturbations

We really want to solve the coupled $N \times N$ system

$$f\frac{d}{dr}\left(f\frac{d\mathbf{\Phi}}{dr}\right) + \left[\omega^2 - f\mathbf{V}\right]\mathbf{\Phi} = 0 \qquad \mathbf{\Phi} = \{\Phi_i\}\ (i = 1, \ldots, N)$$

$$\mathbf{V}(r) = V_{ij}(r)$$

where each matrix element is perturbed:

$$V_{ij} = V_{ij}^{\rm GR} + \delta V_{ij} \qquad \delta V_{ij} = \frac{1}{r_H^2}\sum_{k=0}^{\infty}\alpha_{ij}^{(k)}\left(\frac{r_H}{r}\right)^k$$

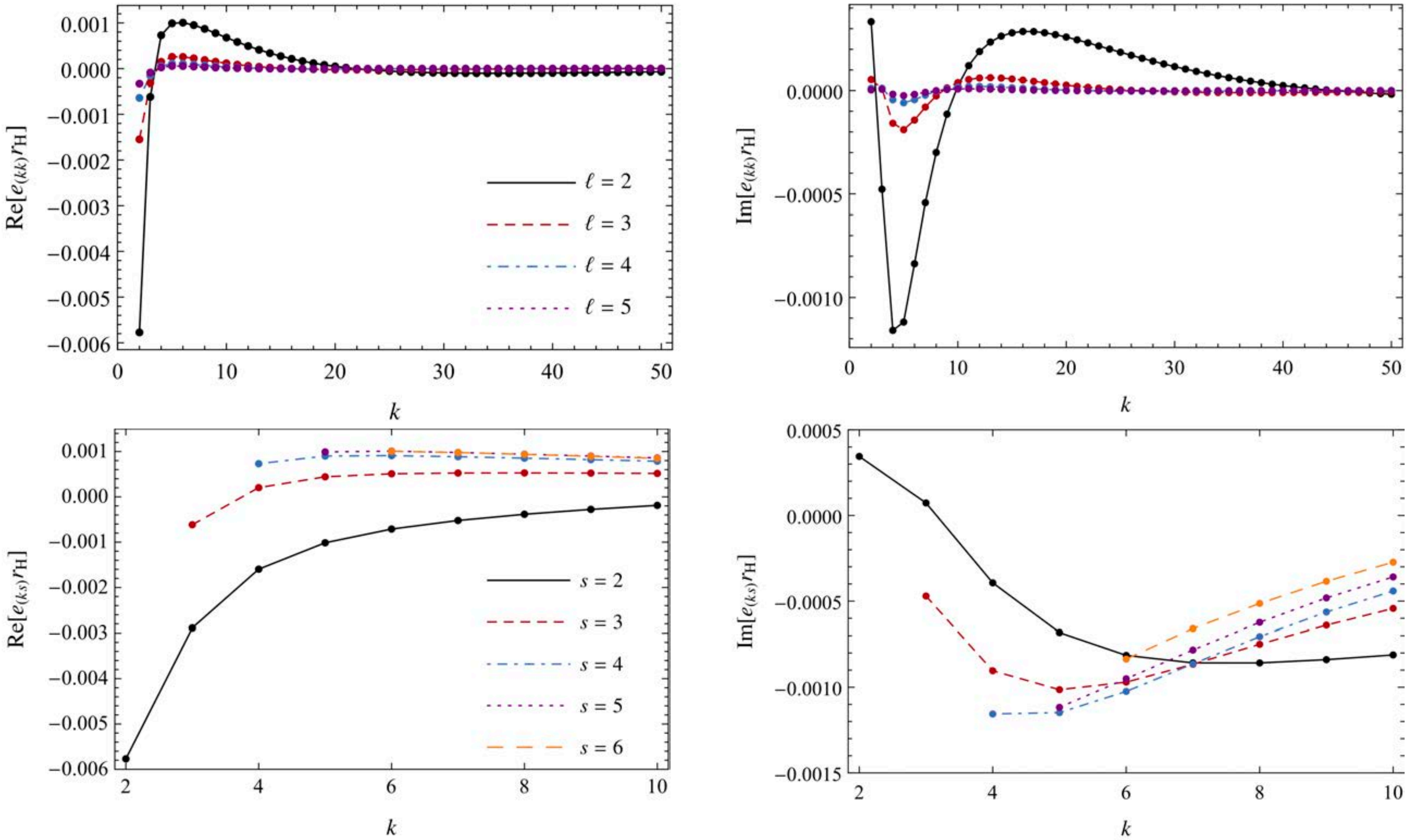
If the background spectra are nondegenerate, coupling will induce **quadratic** corrections.
Allow α to depend on ω. We need

- quadratic corrections in α, besides the linear diagonal terms $d_{(k)}^{ii}$
- coupling-induced corrections

$$\omega \approx \omega_0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \alpha_{pq}^{\prime(s)} d_{(k)}^{ij} d_{(s)}^{pq} + \frac{1}{2}\alpha_{ij}^{(k)}\alpha_{pq}^{(s)} e_{(ks)}^{ijpq} \qquad \text{(Einstein summation)}$$

[McManus+, 1906.05155]

Correction coefficients



The degenerate case

Degenerate spectra (e.g. **even/odd gravitational perturbations**) need special care. Why?

$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0\right)\phi_1+\alpha Z\phi_2=0$$
$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0\right)\phi_2+\alpha Z\phi_1=0$$

Diagonalize:

$$\phi_1=(\phi_++\phi_-)/2$$
$$\phi_2=(\phi_+-\phi_-)/2$$

$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0+\alpha Z\right)\phi_+=0$$
$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0-\alpha Z\right)\phi_-=0$$

Corrections are linear in α

Use degenerate perturbation theory:

$$\omega=\omega_0+\epsilon\omega_1 \qquad \omega_1=\frac{\delta V_{++}+\delta V_{--}\pm\sqrt{(\delta V_{++}-\delta V_{--})^2+4\delta V_{+-}\delta V_{-+}}}{2}$$

$$\delta V_{\pm\pm}=\sum_{k=0}^{\infty}\alpha_{\pm\pm}^{(k)}\left\langle\omega_0,\pm\left|f(r)\frac{r_H^{k-2}}{r^k}\right|\omega_0,\pm\right\rangle=\sum_{k=0}^{\infty}\alpha_{\pm\pm}^{(k)}\delta V_{\pm\pm}^{(k)}$$

Example 1: scalar/odd gravitational in dynamical Chern-Simons

Spectra are nondegenerate

The perturbed potentials read:

$$V_{11} = V_-$$

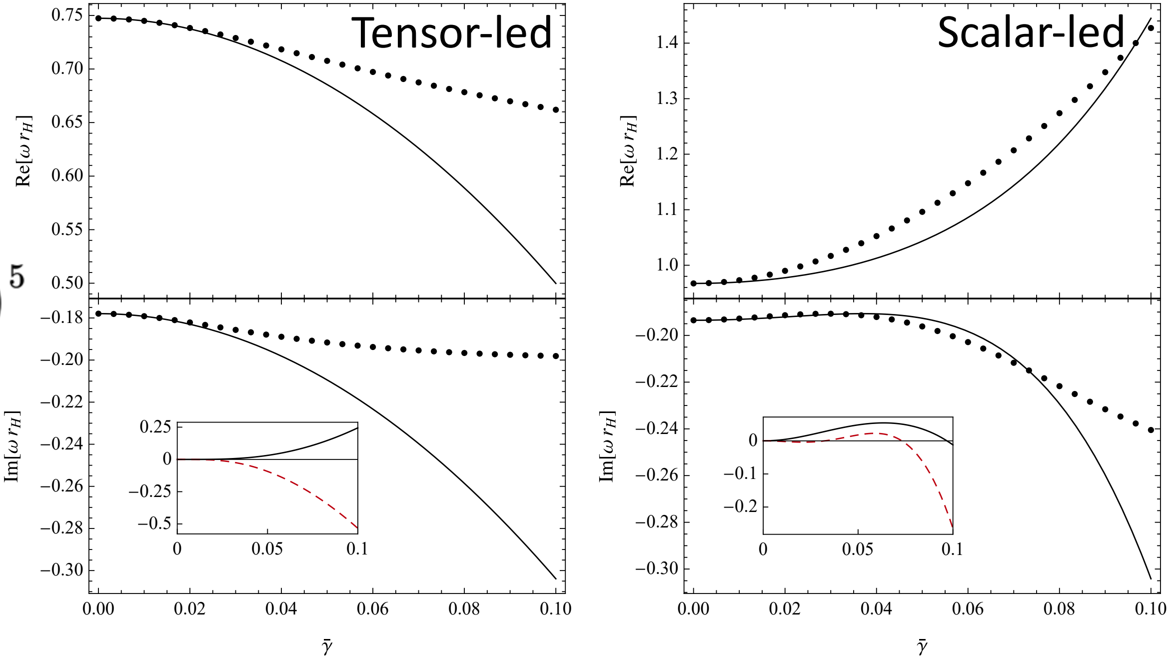
$$V_{12} = V_{21} = \frac{1}{r_H^2}\frac{12}{\sqrt{\beta}r_H^2}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\left(\frac{r_H}{r}\right)^5$$

$$V_{22} = V_{s=0} + \frac{1}{r_H^2}\frac{144\pi\ell(\ell+1)}{\beta r_H^4}\left(\frac{r_H}{r}\right)^8$$

Corrected frequencies:

$$\omega = \omega_0 + e^{1221}_{(55)}\left(12\bar{\gamma}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\right)^2$$

$$\omega = \omega_0 + 2d_{(8)}144\pi\ell(\ell+1)\bar{\gamma}^2 + e_{(88)}\left[144\pi\ell(\ell+1)\bar{\gamma}^2\right]^2 + e^{1221}_{(55)}\left(12\bar{\gamma}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\right)^2$$



[Cardoso-Gualtieri, 0907.5008; Molina+, 1004.4007]

Example 2: scalar-led perturbations in Horndeski

The scalar-led perturbation is related to background coupling functions in the Horndeski Lagrangian:

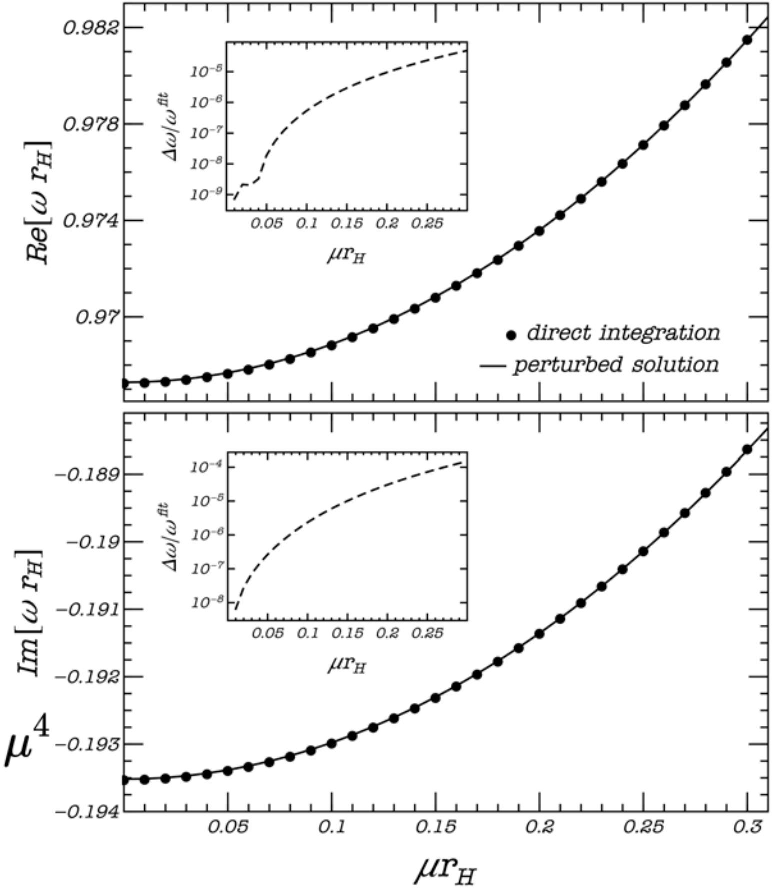
$$\frac{d^2\phi}{dr_*^2} + \left[\omega^2 - f\left(V_{s=0} + \mu^2 + \frac{\ell(\ell+1)}{r^2} f\Gamma\right)\right]\phi = 0$$

$$\mu^2 = \frac{-\overline{G}_{2\phi\phi}}{3\overline{G}^2_{4\phi} + \overline{G}_{2X} - 2\overline{G}_{3\phi}}$$

$$\Gamma = \frac{8\overline{G}_{4X}}{3\overline{G}^2_{4\phi} + \overline{G}_{2X} - 2\overline{G}_{3\phi}}$$

Corrected frequencies read (can set $\Gamma = 0$):

$$\omega \approx \omega_0 + d_{(0)}\mu^2 + \left[d_{(2)} + r_H d_{(3)}\right]\ell(\ell+1)\Gamma + \frac{1}{2}e_{(00)}\mu^4$$
$$+ \frac{1}{2}\left[e_{(22)} + 2r_H e_{(23)} + r_H^2 e_{(33)}\right]\left[\ell(\ell+1)\Gamma\right]^2$$
$$+ \left[e_{(02)} + r_H e_{(03)}\right]\ell(\ell+1)\mu^2\Gamma$$



[Tattersall+, 1711.01992]

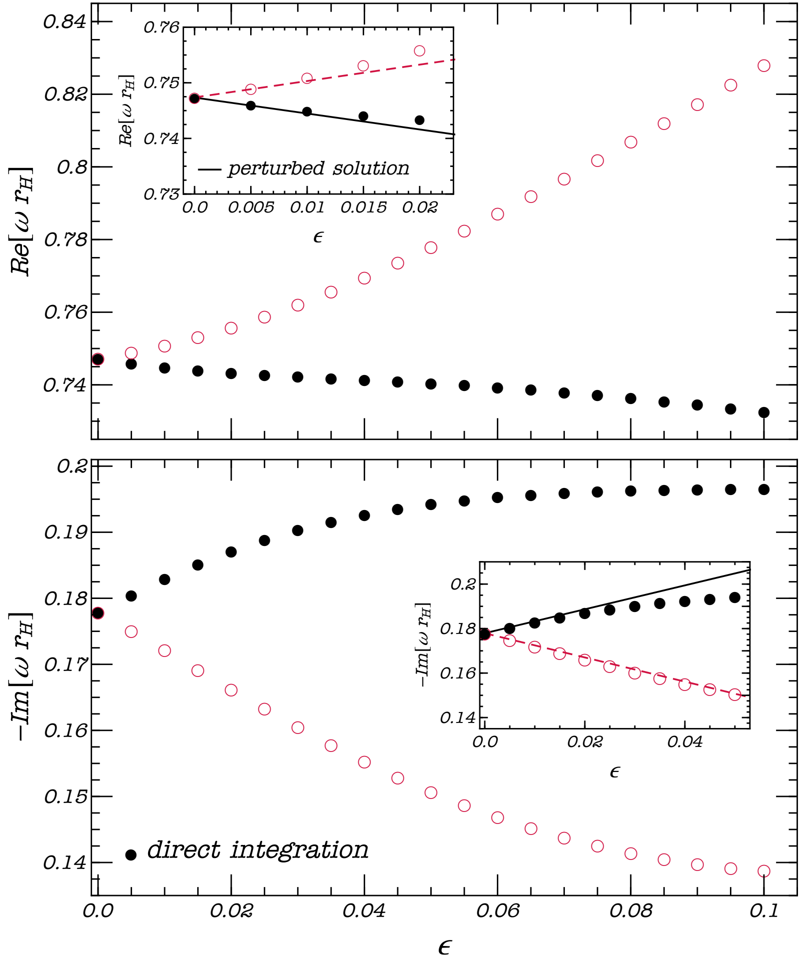
Example 3: odd/even gravitational coupling in EFT (degenerate)

The quartic-in-curvature EFT leads to a degenerate perturbed eigenvalue problem:

$$V_{11} = V_+$$
$$V_{22} = V_-$$
$$V_{12} = V_{21} = \epsilon V(r)$$

where off-diagonal perturbations are given in [Cardoso+, 1808.08962]

Direct integration vs. degenerate parametrization: good agreement, but quadratic corrections could be useful



Parametrized merger/ringdown: a summary

Modifications to the gravity sector and/or beyond Standard Model physics:
- small modifications to the potentials
- coupling between the (matrix-valued) wave equations

We parametrized modifications by power laws, then computed perturbed QNMs for:
- linear corrections to diagonal terms [Cardoso+, 1901.01265]
- quadratic corrections + coupling [McManus+, 1906.05155]

The formalism is very general!
Examples:
- EFT, Reissner-Nordström, Klein-Gordon in Kerr for slow rotation
- scalar/odd gravitational dCS, scalar-led Horndeski, odd/even gravitational EFT

Needed generalizations:
- higher-order corrections (in particular, in degenerate coupled case)
- **coupled gravitational modes with rotation – LIGO/Virgo remnants have spins 0.7 or so!**

Parametrized spectroscopy: adding rotation

Rotating BH QNMs in modified gravity: the EFT viewpoint

QNM calculations: limited sample for specific theories (EdGB/EsGB, dCS) and **nonrotating** BHs
[Blazquez-Salcedo+ 1609.01286 (EdGB), 2006.06006 (EsGB); Molina+ 1004.4007 (dCS)]
Cano's work: systematic small-rotation expansion + **scalar** QNMs
Theories: sum over curvature invariants with scalar-dependent coefficients

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R + \sum_{n=2}^{\infty}\ell^{2n-2}\mathcal{L}_{(n)}\right]$$ and more specifically, at order ℓ^4

EsGB dCS (dilaton+axion)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\Big\{R + \alpha_1\phi_1\ell^2 R_{\rm GB} + \alpha_2(\phi_2\cos\theta_m + \phi_1\sin\theta_m)\ell^2 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$
$$+\lambda_{\rm ev}\ell^4 R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\delta\gamma}R_{\delta\gamma}^{\mu\nu} + \lambda_{\rm odd}\ell^4 R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\delta\gamma}\tilde{R}_{\delta\gamma}^{\mu\nu} - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2\Big\}$$

Einsteinian cubic gravity (+parity-breaking) - causality constraints [Camanho+ 1407.5597]
Next order, no new DOFs [Endlich-Gorbenko-Huang-Senatore, 1704.01590]

$$S_{(4)} = \frac{\ell^6}{16\pi G}\int d^4x\sqrt{|g|}\left\{\epsilon_1\mathcal{C}^2 + \epsilon_2\tilde{\mathcal{C}}^2 + \epsilon_3\mathcal{C}\tilde{\mathcal{C}}\right\} \qquad \mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \quad \tilde{\mathcal{C}} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

[Cano-Ruipérez, 1901.01315; Cano-Fransen-Hertog, 2005.03671. See also work by Hui, Penco…]

No calculation of rotating BH QNMs in modified gravity: the EFT viewpoint

Background solutions:
algorithm to compute small-coupling corrections, up to order 14 in rotation

Scalar QNM calculations: "quasi-separable"
For zero coupling, can be separated in terms of spin-weighted **spheroidal** harmonics

$$\nabla^2\psi = \int_{-\infty}^{\infty} d\omega \sum_{m=-\infty}^{\infty} e^{i(m\phi-\omega t)}\mathcal{D}^2_{m,\omega}\psi_{m,\omega}$$

$$\mathcal{D}^2_{m,\omega} = \mathcal{D}^2_{(0)m,\omega} + \lambda\mathcal{D}^2_{(1)m,\omega}$$

$$\psi_{m,\omega} = \sum_{l=|m|}^{\infty} S_{l,m}(x;a\omega)R_{l,m}(\rho)$$

End of the story: second-order radial ODEs can be cast as wave equations via redefinitions of the radial variable/radial WF, and solved either numerically or via WKB

$$\frac{d^2\varphi}{dy^2} + \left(\omega^2 - V(y;\omega)\right)\varphi = 0$$

Note: not all potentials vanish at the horizon

[Cano-Ruipérez, 1901.01315; Cano-Fransen-Hertog, 2005.03671]

Parametrized spectroscopy: how many observations do we need?

Use a small-spin expansion and add parametric deviations to frequency and damping time
Assume you detect **N** sources, and **q** QNM frequencies for each source

$J = 1, 2, \ldots, q$ modes/source

Order in the spin expansion: need at least 4 or 5 in GR

$$\omega_i^{(J)} = \frac{1}{M_i} \sum_{n=0}^{D} \chi_i^n w_J^{(n)} \left(1 + \gamma_i \delta w_J^{(n)}\right)$$

$i = 1, \ldots, N$ sources

$$\tau_i^{(J)} = M_i \sum_{n=0}^{D} \chi_i^n t_J^{(n)} \left(1 + \gamma_i \delta t_J^{(n)}\right)$$

Expansion coefficients in GR

Small, universal non-GR corrections

How many parameters?
If $\gamma_i = \alpha$ for all sources ,
reabsorb $\gamma_i \delta w^{(n)} \to \delta w^{(n)}$

$$\mathcal{P} = 2(D+1)q \longrightarrow D = 4$$

$q = 1$, $\ell = m = 2$ $\longrightarrow$ $\mathcal{P} = 10$

$q = 2$, $\ell = m = 2$, $\ell = m = 3$ $\longrightarrow$ $\mathcal{P} = 20$

How many observables?

$$\mathcal{O} = 2N \times q$$

$\boxed{\text{Need only } N \geq D + 1}$

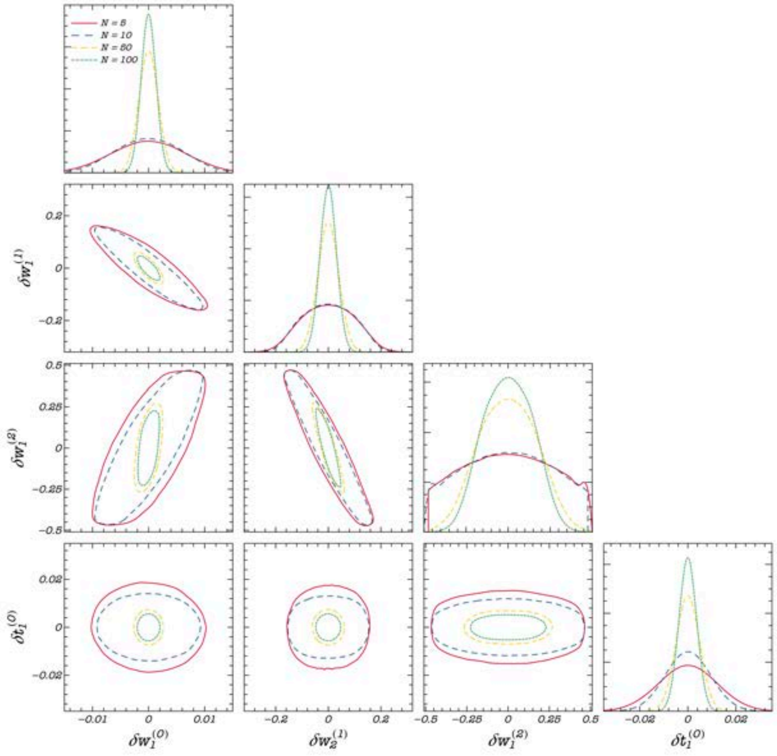
[Maselli+, 1711.01992]

No calculation of rotating BH QNMs in modified gravity: the EFT viewpoint

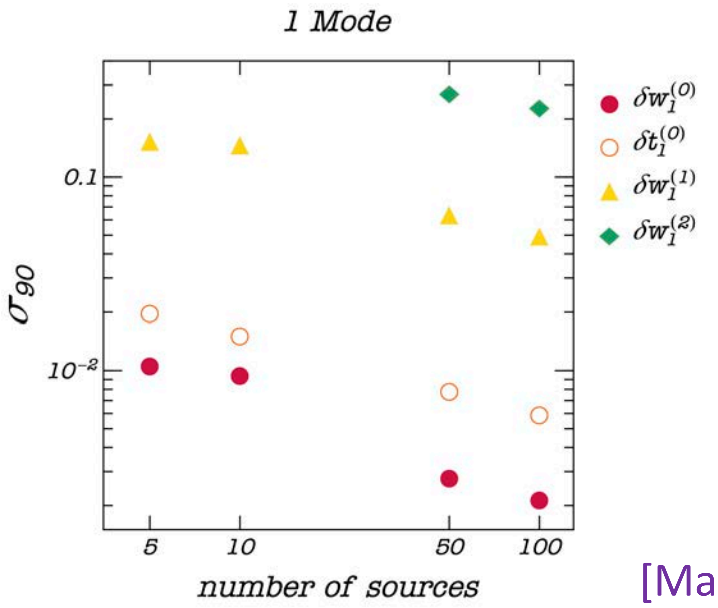
Complication: the coupling is often dimensionful $\gamma_i = \frac{\alpha}{(M_i^{\rm s})^p} = \frac{\alpha(1+z_i)^p}{M_i^p}$

Use Bayesian inference (MCMC), $p=0$, $q=1$ (one mode), simple source distributions

Einstein Telescope: constrain first three frequency coeffs and only the first damping coeffs



Width at 90% confidence gets better as we get more observations:



[Maselli+, 1711.01992]

Take-home messages

“Null” spectroscopic tests of GR:

High-SNR (LISA/CE/ET) GW astronomy will need better control over systematic errors [Ferguson+ 2006.04272]

Study excitation factor with “true” merger initial data. Nonlinearities?

“Real” tests of GR with black hole inspiral/merger/ringdown:

Need full nonlinear simulations in beyond-GR theories

- Identify interesting theories: EsGB is one such
- Study 3+1 decomposition and well-posedness (EFT - or better, full theory)

Parametrized tests of GR with black hole ringdown:

Nonrotating case quite well understood – but irrelevant to most “real” mergers

Rotation:

- Can use perturbation theory and slow-rotation approximation, but hard
- **Numerical, single black hole time-evolutions could get the QNM spectrum in specific theories first** (similar work has been done for rotating stars)